

Teaching Number Sense

The cognitive sciences offer insights into how young students can best learn math.

Sharon Griffin

What is number sense? How does it develop? How can we teach number sense to students who start school without it? Recent research in the learning sciences has found answers to each of these questions. Coming from fields of inquiry that are distinct from education, these answers provide new perspectives on what number sense is and how to teach it.

What Is Number Sense?

Research on teachers' ideas about math (Griffin & Case, 1997) reveals that many teachers define the discipline as *a fixed body of knowledge involving numbers and their manipulation through rules and algorithms* (Jackson, 1986). Because of their own learning experiences, many teachers believe that math is about numbers. By treating numbers as disembodied entities, these teachers focus their instruction on ensuring that their students know various math rules and the applications of those rules.

Although numbers and algorithms are clearly involved in the business of doing math, they are not the whole story—not by a long shot. Instead, mathematicians and enlightened educators see math as a set of conceptual relationships between



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quantities and numerical symbols. This view underpins the *Principles and Standards for School Mathematics* developed by the National Council of Teachers of Mathematics (2000). When teachers see math as about *quantity*, not *numbers*, they ask different questions in their classrooms: What are numbers? instead of What are the rules for manipulating numbers? Students do not simply learn and apply rules and try to find the right answer; instead, they construct and discover relationships between quantities and numbers and then examine alternative ways to describe and record these relationships.

What are the consequences of these two views for student learning? On extensive tests of number knowledge and number sense (Griffin & Case,

1997), kindergarten students who have learned to link numbers to quantities have no trouble answering the question, Which is bigger, 7 or 9? Students without this knowledge often look startled by the question, as if it isn't a meaningful thing to ask, and respond by saying "I don't know" or by making a wild guess. At the end of 1st grade, students who understand the connection between numbers and quantities have no trouble crossing out the three incorrect formulas in the following set of choices: $9 + 2 = 92$; $9 + 2 = 11$; $9 + 2 = 10$; $9 + 2 = 7$. Although some of these students make a mistake in counting and select 10 as the correct answer, they rarely, if ever, select 92 or 7 as the correct answer. In contrast, students who only learn the rules reveal a lack of

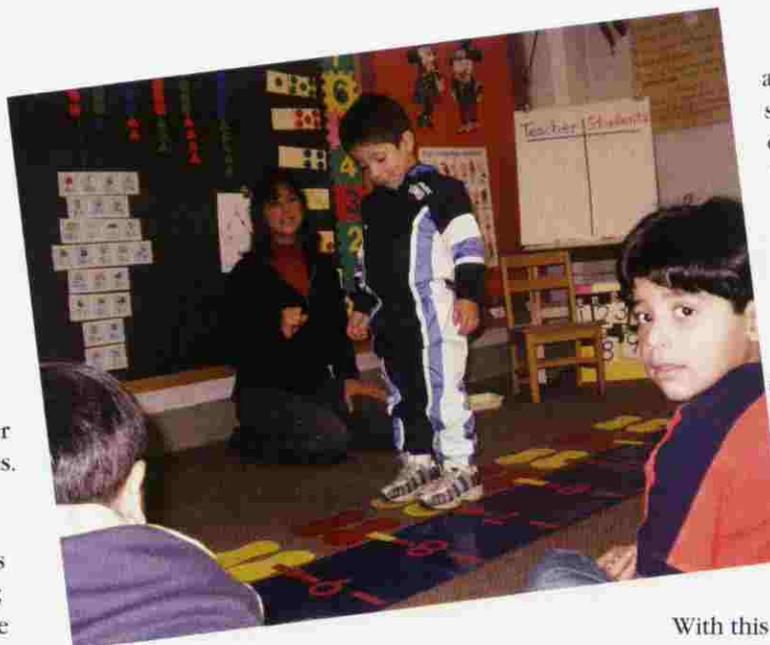
understanding of the meaning of numbers and of the operation signs. Students who choose 92 as the correct answer, for example, are clearly putting numbers together, but they are not making sense of the symbolic expression that asks them to add the quantities that these symbols refer to rather than the symbols themselves.

The discipline of mathematics comprises three worlds: the actual quantities that exist in space and time; the counting numbers in the spoken language; and formal symbols, such as written numerals and operation signs. Number sense requires the construction of a rich set of relationships among these worlds. Students must first link the real quantities with the counting numbers. Only then can students connect this integrated knowledge to the world of formal symbols and gain an understanding of their meaning. To attain number sense, students need opportunities to discover and to construct relationships among these three worlds at higher and higher levels of complexity.

How Does Number Sense Develop?

Pioneering work in cognitive neuropsychology and infant cognition tells us that human infants are born with brain structures that are specifically attuned to numerical quantities. These structures have a long evolutionary history and are at least partially independent of the brain structures that support verbal processing (Dehaene, 1997; Dehaene & Cohen, 1995; Kunzig, 1997).

These structures permit infants, for example, to distinguish a set of two objects from a set of three objects in the first few days of life (Antell & Keating, 1983) and to match a set of three sounds to a set of three objects at six months (Starkey, Spelke, & Gelman, 1990). As early as five months old,



as they merge these two schemas into a single, superordinate conceptual structure for number. This new concept closely connects number with quantity and enables children to use the counting numbers without needing the presence of physical objects to make a variety of quantity judgments, such as determining how many objects they would have altogether if they had 4 of something and received 3 more.

The Number Worlds program enables students to actively explore math concepts, such as position on a number line, in the context of games.

infants can even anticipate the results of transformations in small sets. For example, they register surprise if two puppets are placed behind a screen in sequence and only one is present when the screen is raised, and, conversely, show the same response if one puppet is withdrawn from two that have been placed behind a screen and two puppets are present when the screen is raised (Wynn, 1992). Whether one accepts the strong interpretation of these findings (that infants have an innate ability to represent number and to perform simple arithmetic) or prefers a weaker interpretation (that infants have a remarkable attunement to the magnitude of small sets), it is clear that a strong foundation for number sense is present in the earliest months of life.

As infants become toddlers, their natural quantitative competencies expand and they acquire language and, with it, the ability to count. By the age of 4, as documented in extensive cognitive developmental research (summarized in Griffin & Case, 1997), children have constructed two schemas: one for making global quantity comparisons and another for counting. At age 5 or 6, children experience a revolution in thought

With this new conceptual structure, which researchers believe provides the basis for all higher-level mathematics learning, children have acquired the conceptual foundation for number sense.

Students' conceptual structures continue to develop across the middle childhood years in the manner predicted by Case's (1992) cognitive developmental theory. By the age of 7 or 8, children's central conceptual structure has become more complex, permitting children to make sense of two distinct quantitative dimensions, such as 10s and 1s in math, hours and minutes in time, and dollars and cents in money. With this more complex structure, students are able to understand place value, solve double-digit arithmetic problems ($12 + 54$) in their heads, and tell which of two double-digit numbers (69 or 71) is bigger or smaller. Finally, by the age of 9 or 10, students' more elaborate and integrated central conceptual structure permits them to handle three quantitative variables in a coordinated fashion. With this new structure—the culmination of development in this stage—students acquire a well-developed understanding of the whole number system and are able, for example, to perform mental computations with double-digit numbers that involve borrowing and carrying (such as $13 + 39$) and to solve arithmetic problems involving triple-digit numbers.

An understanding of this developmental progression is important for teaching number sense to all students. Because the progression describes the cognitive capabilities that teachers normally expect from the average child in developed societies, it exerts a profound and perhaps hidden influence on many issues of schooling, such as the age at which students start their formal math instruction and the age and grade level at which teachers introduce particular math concepts. Students who do not conform to this typical pattern may be seriously handicapped in their attempts to make sense of math instruction in school.

students who received this instruction in their kindergarten year demonstrated

- Significant gains in number knowledge, enabling them to start 1st grade on a level commensurate with that of their middle-income peers;

- Significant gains on a variety of transfer tests in such skill areas as time knowledge, money knowledge, and scientific reasoning, demonstrating that they could transfer their knowledge to a broad range of quantitative tasks; and

- Average to above-average performance on a variety of measures in a follow-up study of 1st grade learning and achievement as shown in written arithmetic tests and teachers' ratings of

rich activities for making connections, exploring and discussing concepts, and ensuring an appropriate sequence of concepts.

Rich Activities for Making Connections

The Number Worlds program provides a rich set of activities that expose students to the three worlds of math—quantities, counting, and formal symbols—and to multiple opportunities for constructing relationships among the three worlds. Games expose young students to five different forms of number representation that are common in our culture: groups of objects in Object Land, dot-set patterns and numerals in Picture Land, position on a path or line in Line Land, position on a vertical scale in Sky Land, and position on a dial in Circle Land. Motivated by the desire to win the games, students encounter many opportunities to use numbers to make sense of quantity representations. Expanded programs for four grade levels (preK–2) enable students to forge connections among these worlds at increasing levels of complexity.

The games and props in Line Land, for example, enable students to move their own bodies on a life-size, numbered teddy bear path at the preK level; to move a pawn representing the self along a game-board path, numbered 1–10, at the kindergarten level; to move a magic shoe pawn, which can leap over 10 houses in a single bound, along a game board depicting a row of 100 numbered houses at the 1st grade level; and to move a deliveryman pawn up floors and along corridors on a hotel game board with 100 numbered rooms stacked in floors of 10 units each at the 2nd grade level. Because the number system is fixed in these spatial environments and displayed in a prominent fashion, students playing the games can discover important properties of this system.

The games' rules require students to count out loud as they move through these spatial environments, ostensibly so

When teachers see math as about quantity, not numbers, they ask different questions in their classrooms.

Because higher-level conceptual structures depend on the core concepts that students typically acquire at the age of 5 or 6, students whose core structure is not in place at the expected age will experience serious delays and will have difficulty catching up with their peers. Although most children acquire these developmental milestones at the expected age, a substantial number of students in the United States—typically those living in low-income communities—start school at the age of 5 or 6 without the central conceptual structure in place. On developmental tests of number knowledge, their performance is often nearly two years below the level of their middle-income peers (Griffin, Case, & Siegler, 1994).

We Can Teach Number Sense

A research-based mathematics program called Number Worlds (Griffin & Case, 1996), originally called Rightstart, has tested this developmental theory and several methods for teaching the central concepts that underlie number sense. In a series of studies conducted over several years, at-risk populations of

number sense (Griffin, Case, & Siegler, 1994).

In contrast, the control groups of at-risk students whom we followed in these studies, who participated in a variety of other math programs, continued to underperform on all measures. Although they made some progress in kindergarten and 1st grade, the developmental lag that had been present at the beginning of kindergarten was still evident on measures of math learning and achievement administered at the end of 1st grade.

The Number Worlds program's success not only provides support for the psychological theory on which it was based but also suggests that teaching number sense is possible and that certain instructional principles drawn from recent theory and research on how people learn (Bransford, Brown, & Cocking, 1999; Griffin, in press) provide a powerful set of tools to teach it.

How to Teach Number Sense

Three instructional principles lie at the heart of teaching number sense and the Number Worlds program: providing

that other players can be sure that they don't cheat. In the process of counting, predicting, and explaining their movements, students become proficient at using the standard linguistic terms—such as the language of distance—for describing quantities in this context. Most of the 1st and 2nd grade games also require students to create a written record of their actions, using the formal

understandings to construct new understandings at the next level up. Activities in the preK program enable 4-year-old students to use their conceptual structures to acquire the foundational knowledge that they will need later. Activities in the kindergarten program allow students to construct and consolidate their central conceptual structure. Within each grade level, activities have

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symbol system to record their movements and transactions to help determine the winner or who landed on a secret number. Other games and activities serve the same purpose and also expose students to other ways of representing number 5, such as in a dot-set pattern or a position on a scale.

Exploration and Discussion

The program provides opportunities for students to actively explore the concepts and to discuss them in a social context. The game formats require small groups of four to five students to work collaboratively to achieve the game goal. Each child actively participates by taking his or her turn, moving through the environments as the games dictate, and describing his or her movements orally or in writing. Students participate enthusiastically because the games are fun and they want to win. The games' rules, the dialogue prompts provided in the teacher's guide, the question cards included with the game props, and the wrap-up period at the end of each lesson all foster mathematical communication.

Appropriate Sequence of Concepts

The program provides a carefully graded sequence of activities that enable students to use their current

also been classified at three levels of developmental complexity. A seamless sequence of activities permits individual students to start at an appropriate individual level and to move through the normal developmental progression at a suitable pace.

What the Learning Sciences Teach

The cognitive sciences have helped us understand that in the course of development, quantity and number become solidly interconnected in children's thought around the age of 5, providing a foundation for number sense and for successful learning of arithmetic. Students who are at risk for school failure often have not achieved this developmental milestone when they start 1st grade, making it difficult for them to make sense of the math taught in school.

Fortunately, the cognitive sciences have also shown us that we can teach number sense to students who arrive at school without it by using an instructional program that is solidly grounded in recent research on how students learn and think. ■

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