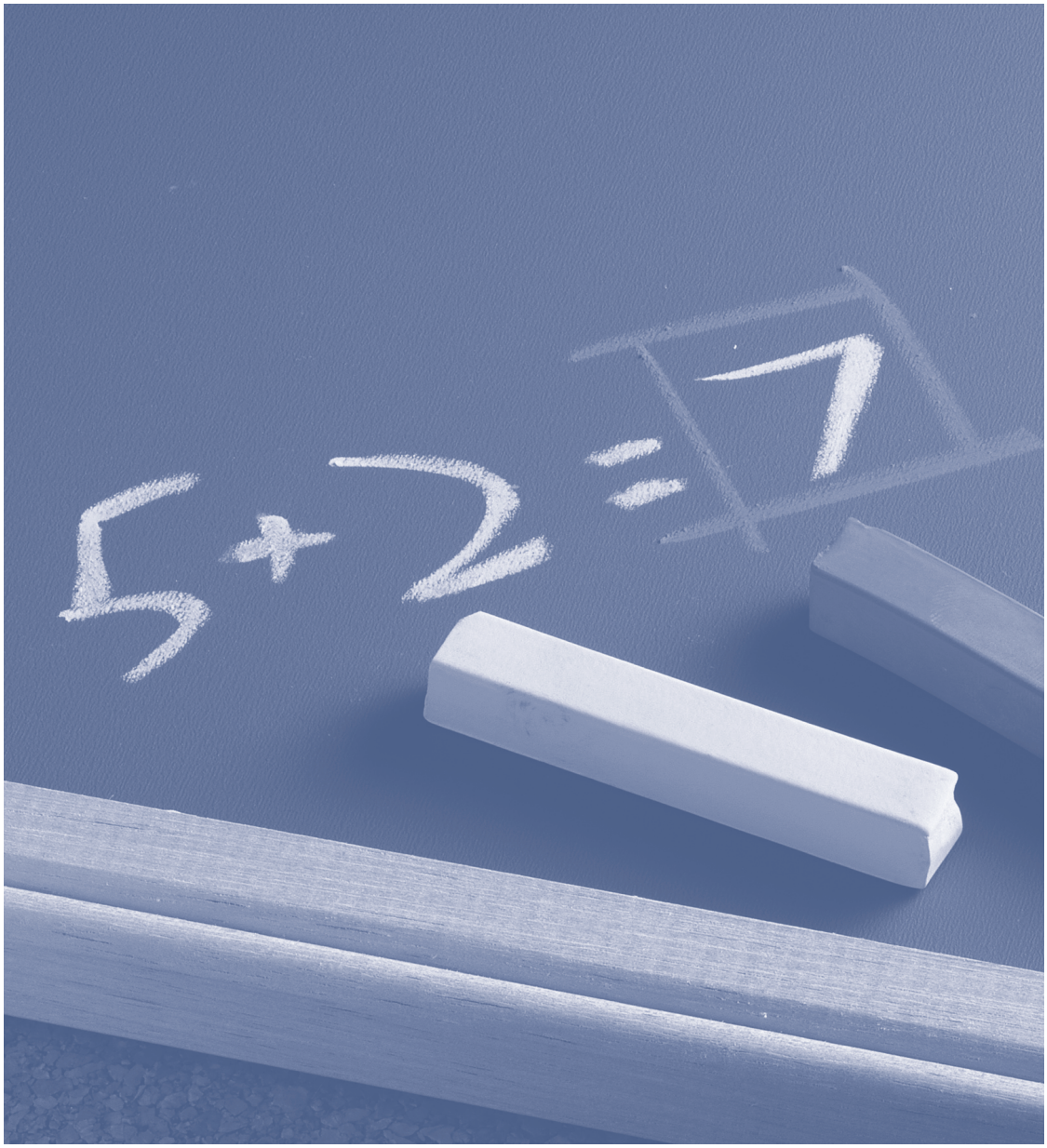


FOUNDATIONS OF MATHEMATICS

PARTICIPANT MANUAL | 2012

PUBLIC SCHOOLS OF NORTH CAROLINA
State Board of Education | Department of Public Instruction :: Exceptional Children Division

Agenda

Foundation Training in Mathematics
Day 1

AGENDA			
WHAT (content)	**HOW (process)**	**WHO (facilitator)**	**TIME (minutes)**
History of the Project Agenda	Presentation Review Discuss Questions & Answers	Math Foundations Trainers	8:30-8:45
Introductions Get to Know You Activity	**Bingo**	Math Foundations Trainers	8:45-9:15
Unit 1 Introduction to the Foundations of Math How to use the Media Products	Presentation Review Discuss	Math Foundations Trainers	9:15-10:15
Break			10:15-10:30
Survey of Elementary Teachers of Mathematics Required Readings for Units 1 and 2	Complete survey Begin Liping Ma Reading	Math Foundations Trainers	10:30-11:45
Lunch			11:45-1:00
Unit 2 Research		Math Foundations Trainers	1:00-2:15
Break			2:15-2:30
Research		Math Foundations Trainers	2:30-4:00
Required Reading Ahead of time: "Reform by the Book" (Deborah Loewenberg Ball / David K. Cohen) "Understanding Math Anxiety" (Sean Cavanaugh) "Teaching Number Sense" Griffin	"Magical Hopes" Ball "Arithmetic in American Mathematics Education" Ma "The Components of Number Sense: An Instructional Model for Teachers", Val Faulkner Read your Chapter of Liping Ma (Subtraction, Multiplication, Division of Fractions, Area and Perimeter)		
Assignments Bring a Lesson Plan to Day 1	Discussion Question for Unit 2 (Discuss in Groups on Day 2)	.	

Foundation Training in Mathematics
Day 2

AGENDA			
WHAT (content)	**HOW (process)**	**WHO (facilitator)**	**TIME (minutes)**
Unit 3 Demystifying Math Part 1 and 2		Math Foundations Trainers	**8:30-10:00**
Break			**10:00-10:15**
Unit 3 Demystifying Math Part 3 and 4		Math Foundations Trainers	**10:15-11:45**
Lunch			**11:45-1:00**
Unit 4 Components of Number Sense		Math Foundations Trainers	**1:00-2:15**
Break			**2:15-2:30**
Unit 4 Components of Number Sense		Math Foundations Trainers	**2:30-4:00**
Reading Assignments	"Subitizing" by Doug Clements Continue reading Liping Ma Text		
Assignment	None		

Foundation Training in Mathematics
Day 3

AGENDA			
WHAT (content)	**HOW (process)**	**WHO (facilitator)**	**TIME (minutes)**
Unit 5 Quantity/Magnitude and Numeration		**Math Foundations Trainers**	**8:30-10:00**
Break			**10:00-10:15**
Unit 5 Quantity/Magnitude and Numeration		**Math Foundations Trainers**	**10:15-11:45**
Lunch			**11:45-1:00**
Unit 6 Base Ten Equality and Form of a Number		**Math Foundations Trainers**	**1:00-2:15**
Break			**2:15- 2:30**
Unit 6 Base Ten Equality and Form of a Number		**Math Foundations Trainers**	**2:30-4:00**
Reading Assignments	None		
Assignment	Discussion Questions for Unit 5 (Discuss in Groups on Day 4) Discussion Question for Unit 6	Learning task 5 (Structures) Learning Task 6 (Number Knowledge Test)	

Foundation Training in Mathematics
Day 4

AGENDA			
WHAT (content)	**HOW (process)**	**WHO (facilitator)**	**TIME (minutes)**
Proportional Reasoning		**Math Foundations Trainers**	**8:30-10:00**
Break			**10:00-10:15**
Proportional Reasoning		**Math Foundations Trainers**	**10:15-11:45**
Lunch			**11:45-1:00**
Algebraic		**Math Foundations Trainers**	**1:00-2:15**
Break			**2:15-2:30**
Algebraic Thinking		**Math Foundations Trainers**	**2:30-4:00**
Required Readings	**"Mathematics Education in the 21st Century" Deborah Ball**		
Assignments	**Learning Task 7 (Painting Walls)**		

Foundation Training in Mathematics

Day 5 _____

AGENDA			
WHAT (content)	**HOW (process)**	**WHO (facilitator)**	**TIME (minutes)**
Unit 7 Geometric Thinking		**Math Foundations Trainers**	**8:30-10:00**
Break			**10:00-10:15**
Unit 7 Geometric Thinking		**Math Foundations Trainers**	**10:15-10:45**
Survey of Elementary Teachers of Mathematics		**Math Foundations Trainers**	**10:45 – 11:45**
Lunch			**11:45 – 1:00**
Unit 8 Assessment		**Math Foundations Trainers**	**1:00 -2:15**
Break			**2:15 – 2:30**
Unit 9 Connections Evaluation		**Math Foundations Trainers/Participants**	**2:30-4:00**
Assignments		**Complete Learning Task 9 in groups.**	

Learning Tasks

Math Foundations Trainer Requirements Checklist for Responsibilities

1. Completion of Level 1 requirements earns 4 CEUs.

Requirements include:

- **100% attendance, participation in all 5 days.**
- **Study readings & respond to discussion questions assigned for Level 1 training.**
- **Completion of all learning tasks assigned for Level 1 training.**
- **Successful completion of submitted homework acknowledged by trainers.**

2. Completion of Level 2 requirements earns 3 CEUs.

Requirements include:

- **100% attendance, participation in all 5 days (one of the two training weeks must be training sponsored by DPI).**
- **Completion of all assignments and homework acknowledged by trainers.**

3. Assessment of conceptual math knowledge.

Requirements to become a Math Foundations Trainer include:

- **Aspiring trainers are responsible for documenting completion of Levels 1, 2 and all other trainer requirements. Documentation examples include: e-mails, signatures.**
- **Schedule math trainer assessment with DPI within 1 year of completion of Level 2 training by contacting the DPI Math SIP Coordinator.**
- **Pass the trainer assessment with 80% accuracy (PRIOR to scheduling any training in LEA).**
- **Send training dates to DPI six weeks PRIOR to training (to allow time for scheduling of observations).**
- **Team train during first training with a satisfactory evaluation by a master trainer.**
- **Complete three required observations. Two of these observations are scheduled during training days 2, 3, or 4.**
- **The final observation is submitted as a video tape with self-reflection completed. Directions and forms for the self-reflection will be provided when trainee is at this point.**
- **Submit documentation of all completed requirements to the LEA SIP Coordinator and DPI Math Sip Coordinator to be able to:**
- **Attend and receive certification at Training of Trainers event.**
- **Celebrate this accomplishment!**

*****Principal and EC Director Approval (signature) are required to become a Math Foundations Trainer.**

Level 1 Learning Tasks –Minimum Rubrics Score

Learning Task 5 Structures	Learning Task 6 Number Knowledge Test	Learning Task 7 Painting Walls	Learning Task 9 Revised Lesson Plan
9 out of 12	12 out of 16	12 out of 16	10 out of 12

Level 2 Learning Task—Minimum Rubrics Score

Learning Task 2 PowerPoint	Learning Task 5 Griffin Lesson Plan	Learning Task 6/8 NKT/Assessment
14 out of 16	11 out of 12	14 out of 16

Level I

Math Foundations Level I Assignments

All Discussion Questions will be used to focus a particular day.

All Learning Tasks are to be completed individually with the exception of Learning Task 9, which will be a group effort.

Learning Tasks (LT 5, 6 and 7-Complete Individually)

Learning Task for Unit 5 (See Rubric)

Recall the four structures of addition, subtraction, multiplication and division.

Addition
- **Join and Part Whole**
- **Start Unknown**
- **Compare-Total Unknown**

Subtraction
- **The Classic "Take away" (how many left?)**
- **Comparison (difference between? who has more?)**
- **Deficit/Missing amount (what's missing?)**

Multiplication
- **Repeated Addition**
- **Array/Row-Column**
- **Counting Principle**

Division
- **Measurement/Repeated Subtraction**
- **Partitive/Unitizing-Fair Shares**
- **Product/Factor**

Directions

For each Operation and each Structure:

Write a simple word problem that matches the structure.
Keep the values small for simplicity.
Diagram the structure using a graphic organizer or tool. (bar diagram or number line)
Include a number sentence for each structure.
(i.e. 2 cupcakes + 5 cupcakes = 7 cupcakes)

You will have 12 diagrams and 12 word problems.

Learning Task for Unit 6 (See Rubric)

Select a student who is struggling in mathematics as compared to his/her peers. Administer the Number Knowledge Test, annotating the recording sheet diligently. Make certain to read and refer to the Directions for Administration for help in administering and interpreting the Number Knowledge Test. Provide a background of the student, Analyze the results of the test to determine the student's strengths and weaknesses. Include two or three possible interventions based on strategies you have learned in Foundations of Math.

Learning Task 7 (See Rubric)

Proportional Challenge

***Painting Walls* is a classic proportional reasoning problem. It was used with prospective teachers on a final exam.**
("Prospective Elementary School Teachers' Solution Strategies and Reasoning for a Missing Value Proportion Task", Jane-Jane Lo, Western Michigan University 2004)

Jane can paint a 12-foot by 12-foot wall in ¾ of an hour?
How long will it take her to paint another wall that is 15 x 16 feet?

Do not solve the problem by the use of cross-multiplication.

Draw pictures to illustrate the situation in the problem.

Explain how you can use your pictures to justify your solution.

Unit 9 – Learning Task (See Rubric) (Completed in Groups)

- **Revisit your lesson plan or one that has been provided by your trainer.**
- **Choose one lesson plan per table to discuss as a group.**
- **Using your rubric in your handout section, assess the initial lesson plan with a rubric score. Record the scores in Column 1.**
- **Discuss as a group how you can make the initial lesson plan stronger as it relates to the content of this training.**
- **As a group, list five changes you could make to the lesson plan to strengthen the plan in terms of learning for the student. All five changes should not come from one area of learning from Foundations of Math.**
- **What would be your new rubric score with the changes added? Record those scores.**
- **On a sheet of paper, list the names of the participants in your group. Turn in the lesson plan, list of five changes, and the pre and post rubric scores for each of the two lessons as well as a brief group reflection on the two lesson plans.**

Readings for Foundations of Math

The articles must be read by the specified day listed below.

Day 1 (Read Prior to Day 1)

Unit 1

"Reform by the Book" Ball

Unit 2

"Teaching Number Sense", Sharon Griffin

"Understanding Math Anxiety", Callahan

Day 2 (Read Prior to Day 2)

Unit 3

"Magical Hopes" Deborah Ball

Assigned Chapter of Knowing and Teaching Elementary Mathematics, Liping Ma

Unit 4

"Arithmetic in American Mathematics Education: An Abandoned Arena?" Liping Ma

"The Components of Number Sense: An Instructional Model for Teachers", Val Faulkner

Day 3 (Read Prior to Day 3)

Unit 5

"Subitizing" by Doug Clements

"Singapore Math" by John Hoven and Barry Garelick

Day 5 (Read Prior to Day 5)

Unit 9

"Mathematics Education in the 21st Century" Deborah Ball

Discussion Questions

(Be prepared to discuss in groups if time permits. You do not need to write out your answers.)

Discussion Question for Unit 2 (Discuss in Groups on Day 2)

> **Reflect on your own teaching and discuss how each of the findings of the National Math Panel will impact your own instruction in the classroom or in the way in which you will assist teachers.**

Discussion Questions for Unit 5 (Discuss in Groups on Day 4)

> **Compare the instruction that you have seen that is guided by Hands-on-Equations to other math instruction that you have seen at the secondary level for early algebra.**

Discussion Question for Unit 6 (Discuss in Groups on Day 4)

> **Discuss how your understanding regarding base ten, equality/inequality and form of a number changed after exposure to this unit?**

TELT-Teacher Education and Learning to Teach **Passing Score -12/1** **Foundations of Math**

Participant Name(s) **Rubric**

CATEGORY	4	3	2	1	Category Score
Subtraction (Key Ideas) Decomposing and Composing a Unit of Higher Value Unit (Base Ten System, Rate and Equal Exchange); Facts within 20; Multiple ways of Regrouping; Forms of the Value; Knowledge Package	Reflection includes a reference to four Key Ideas	Reflection includes a reference to three Key Ideas	Reflection includes a reference to two Key Ideas	Reflection includes a reference to one Key Idea.	(/4)
Multiplication (Key Ideas) Distributive Law; Place Value as a System; Base Ten; Partial Products; Form of the Value and Flexibility of Mental Math; Concrete Multiplication; Knowledge Package	Reflection includes a reference to four Key Ideas	Reflection includes a reference to three Key Ideas	Reflection includes a reference to two Key Ideas	Reflection includes a reference to one Key Idea.	(/4)
Division of Fractions (Key Ideas) Conceptual understanding via story Problems; Understanding the Algorithm; Understanding through Proportional Reasoning; Flexible Forms for Computation; Relatedness of Subtraction and Division; Idea of Unit size; Properties (Multiplicative Identity and Maintaining the Value of the Quotient); Knowledge Package	Reflection includes a reference to four Key Ideas	Reflection includes a reference to three Key Ideas	Reflection includes a reference to two Key Ideas	Reflection includes a reference to one Key Idea.	(/4)
Area and Perimeter (Key Ideas) Disproving the Claim; Identifying the Possibilities; Clarifying Conditions; Explaining the Conditions; Knowledge Package; Distributive Law; Mathematical Attitude and Knowledge of Topic	Reflection includes a reference to four Key Ideas	Reflection includes a reference to three Key Ideas	Reflection includes a reference to two Key Ideas	Reflection includes a reference to one Key Idea.	(/4)

Total **(/16)**

Structures of Addition, Subtraction, Multiplication and Division

Participant Name Learning Task 5 Rubric

CATEGORY	4	3	2	1	Category Score
Mathematical Concepts	Explanation shows complete understanding of the mathematical structures used to solve the problem(s).	Explanation shows substantial understanding of the mathematical structures used to solve the problem(s).	Explanation shows some understanding of the mathematical structures used to solve the problem(s).	Explanation shows very limited understanding of the mathematical structures used to solve the problem(s).	(/4)
Alignment of Word Problems	All four of the operations are completed and match the three structures for each operation.	All but one of the operations are completed and match the three structures for each operation. All four are completed and mostly match the structure intended.	Two of the operations are completed and match the three structures for each operation. More than two are completed, but do not match the structure intended.	One of the operations is completed and matches the three structures for each operation. More than one of the operations is completed, but do not match the structure intended.	(/4)
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding for all of the structures.	Diagrams and/or sketches are clear and greatly add to the reader's understanding for most of the structures.	Diagrams and/or sketches are somewhat difficult to understand, but seem to match the structures.	Diagrams and/or sketches are somewhat difficult to understand, and do not match the structures.	(/4)

Total Passing =(9/12) (/12)

Number Knowledge Test

Participant Name | Learning Task 6 | Rubric

CATEGORY	4	3	2	1	Category Score
Background Information	A full background of the student was provided. Student chosen has a skill level appropriate for assessment with the NKT.	Some background of the student was provided. Student chosen has a skill level appropriate for assessment with the NKT.	Little background of the student was provided. Student chosen has a skill level appropriate for assessment with the NKT.	No background of the student was provided. Student chosen does not have the skill level appropriate for assessment with the NKT.	(/4)
Data Collection (Recording Sheet)	All appropriate data was collected on the recording sheet. Detailed notations were made to assist in the interpretation of the data.	All appropriate data was collected on the recording sheet. Some notations were made to assist in the interpretation of the data.	Some data was collected on the recording sheet. Some notations were made to assist in the interpretation of the data.	The data recording sheet was incomplete with few or no notations made to assist in the interpretation of the data.	(/4)
Analysis of Weaknesses and Strengths	Weaknesses and Strengths of the student were clearly stated. A well-aligned valid analysis was developed based on the purpose of the assessment and the data recorded.	Weaknesses and Strengths of the student were stated. Some effort was made in making a valid analysis based on the purpose of the assessment and the data recorded.	Weaknesses or Strengths of the student were stated. Little to no effort was made in making a valid analysis based on the data recorded. Student weaknesses were attributed to spurious issues rather than mathematics performance as documented by the assessment.	Weaknesses or Strengths of the student may or may not be clearly stated. No effort was made in making a valid analysis based on the data recorded.	(/4)
Instructional Recommendations: Instructional Methods; Instructional Materials and Methods for Monitoring	Clear and specific recommendations for Instructional interventions were suggested. Interventions were aligned with documented weakness and supported by research.	Somewhat specific recommendations for Instructional interventions were suggested. Interventions were supported by research.	Recommendations were made for instructional Interventions. Recommendations were not clearly aligned with documented weakness and/or not supported by research.	No recommendation was made for Instructional interventions.	(/4)

Total | Passing = (12/16) | (/16)

Painting Walls

Participant Name | Learning Task 7 | Rubric

CATEGORY	4	3	2	1	Category Score
Mathematical Reasoning	Uses complex and refined mathematical reasoning.	Uses effective mathematical reasoning	Some evidence of mathematical reasoning.	Little evidence of mathematical reasoning.	(/4)
Explanation	Explanation is detailed and clear.	Explanation is clear.	Explanation is a little difficult to understand, but includes critical components.	Explanation is difficult to understand and is missing several components OR was not included.	(/4)
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the problem.	Diagrams and/or sketches are clear and somewhat helps the reader 's understanding of the problem.	Diagrams and/or sketches are somewhat difficult to understand as and may not add to the reader's understanding of the problem.	Diagrams and/or sketches are difficult to understand and do not add to the reader's understanding of the problem.	(/4)
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	More than 75% of the steps and solutions have mathematical errors.	(/4)
Total				Passing = (12/16)	(/16)

Final Lesson Revision

Participant Name(s) **Learning Task 9** **Rubric**

CATEGORY	4	3	2	1	Category Score
Rubrics for the Lesson Plans	Both lessons have a rubric score and are accurately analyzed.	Both lessons have a rubric score and are somewhat accurately analyzed.	Both lessons have a rubric score, but the scores do not accurately reflect the changes in the lesson.	Rubric scores are present, but there was little effort to align them to an appropriate score.	(/4)
Changes included the use of: (The Components of Number Sense), (Language, Cognition, Mathematical Structure) and (Connection to Prior Learning)	Lesson has five valid changes. There is strong evidence of change with respect to all three categories listed.	Lesson has four valid changes. There appears to be evidence of change with respect to all three categories listed.	Lesson has at least three valid changes. There is evidence of change with respect to at least two categories listed.	Lesson has less than three valid changes. There is some evidence of change with respect to some of the categories listed.	(/4)
Reflection on the two Lessons	Reflections about the two lessons strongly emphasize the growth made in the areas of the Components of Number Sense, Mathematical Language and Structure and Connections	Reflections about the two lessons discuss the growth made in the areas of the Components of Number Sense, Mathematical Language and Structure and Connections	Reflections about the two lessons discuss the growth made in most of areas of the Components of Number Sense, Mathematical Language and Structure and Connections	Reflections about the two lessons somewhat discusses the growth made in the areas of the Components of Number Sense, Mathematical Language and Structure and Connections	(/4)

Total **Passing = (9/12)** **(/12)**

Level II

Math Foundation Level II assignments:

Learning Task for Unit 2

Please visit the following site:
http://www.ed.gov/about/bdscomm/list/mathpanel/index.html
Using this site and the summary brochure (Summary of Math Panel report "Foundations for Success"), prepare a PowerPoint presentation from the information regarding the findings of the Math Panel.
In lieu of a PowerPoint of the findings of the National Math Panel, develop a presentation on the key ideas of Foundations of Math.

Present the presentation at a faculty meeting or similar get together.

You should turn in your PowerPoint presentation along with a sign in sheet of the participants.

NOTE: Learning Task 2 can been done in groups. (Four or fewer participants)

The following Learning Tasks must be done individually.

Learning Task for Unit 5

Think of a lesson that you have recently taught or seen taught.
What teaching strategies of Sharon Griffin were used in the lesson? (Quantity, Math Structure, Symbolic) Which teaching strategies from these three areas were missing from the lesson?
How could you adjust this lesson to make certain all teaching strategies were incorporated?
Submit:

1. The original lesson in Word format. Highlight in yellow any of the Griffin strategies that are included in this lesson.

2. In the same Word document, insert any appropriate changes you would make to the original lesson that supports the Griffin model. Highlight these changes in green. Remember that these changes would include research-based practices learned in Foundations of Math.

Learning Task for Unit 6

Further assess the student you assessed with the Number Knowledge Test. If that student is not available, administer the Number Knowledge Test to a different struggling student. Using the data from the Number Knowledge Test, determine where you need more information about the student. Use any further assessment tool such as the Math Instruction Checklist, probes from Intervention Central or teacher made assessments to gain more insight into the student's areas of need. Determine where instruction should begin for this student. In Unit 8, you will use this beginning point to create an initial learning plan for your student.

Learning Tasks Unit 8:

Use the Number Knowledge Test results from the child who you felt was struggling in mathematics (Unit 5). Use the information provided from the further assessment that you did with the student to create an initial learning plan with a beginning goal, objective and a couple of structured learning activities.

Keep in mind that the learning plan needs to cover a few days to no more than a week at a time. We must consider if this short-term plan is working before we continue with the initially planned instruction. Include in this plan how you will monitor progress with data and what further assessment information might be needed? A suggested template has been provided for your convenience.

PowerPoint on the NMP/FofM

Participant Name(s) Level 2 Learning Task 2 Rubric

CATEGORY	4	3	2	1	Category Score
Sequencing of Information; Length of Presentation; Sign-in Sheet	Information is organized in a clear, logical way. It is easy to anticipate the type of material that might be on the next slide. There are between 8 and 12 slides. Sign-in Sheet is included with signatures of participants.	Most information is organized in a clear, logical way. One slide or item of information seems out of place. There are less than 8 or more than 12 slides. Sign-in Sheet is included with signatures of participants.	Some information is logically sequenced. An occasional slide or item of information seems out of place. There are less than 8 or more than 12 slides. Sign-in sheet is included with names of participants, but not signatures.	There is no clear plan for the organization of the information. There are an inappropriate number of slides in the presentation. There is no sign-in sheet.	(/4)
Formatting (Graphics, Background, Spelling and Grammar, Font Choice)	Presentation is attractive and the background, graphics and font formats do not detract from the overall presentation.	Presentation is somewhat attractive and the graphics, background and font formations do not detract from the overall presentation.	All graphics are attractive, but do not seem to support the content of the presentation. The background does not detract from the text. The font may be a little difficult to read.	Presentation has several graphics that detract from the presentation or a font and background that make it difficult to view the text or graphics.	(/4)
Spelling and Grammar	Presentation has no misspellings or grammatical errors.	Presentation has 1-2 misspellings but no grammatical errors.	Presentation has 1-2 grammatical errors but no misspellings	Presentation has more than 2 grammatical errors and/or spelling errors.	(/4)
Content-Accuracy of the National Math Panel Findings/Foundations of Math Training	The Presentation has accurate and key ideas that summarized the key findings of the National Math Panel/Foundations of Math Training. At least five key ideas are highlighted and explained.	Most of the content is accurate, but there is one piece of information that might be inaccurate. At least four key ideas are highlighted and explained from the findings of the National Math Panel/Foundations of Math Training.	The content is generally accurate, but some if the information is irrelevant. At least least three key ideas are highlighted and explained from the findings of the National Math Panel/Foundations of Math Training.	The content contains more than one factual error Only two key ideas are highlighted and explained from the findings of the National Math Panel/Foundations of Math Training.	(/4)

Total

Passing Score 14/16 (/16)

Griffin Lesson Revision

Participant Name(s) Level 2 Learning Task 5 Rubric

CATEGORY	4	3	2	1	Category Score
Appropriate Lesson Plan with Sharon Griffin Components (Quantity, Math Structure, Symbolic)	Original Lesson Plan is included with all of the Griffin Strategies identified and highlighted in yellow.	Original lesson plan is included with most of the Griffin Strategies identified and highlighted in yellow.	Original lesson plan is included, but the Griffin strategies are not identified.	No lesson plan is included.	(/4)
Valid Changes to the Griffin Plan (Quantity, Mathematical Structure and Symbolic)	Lesson has at least five valid changes. There is strong evidence of change with respect to all three categories listed.	Lesson has at least four valid changes. There appears to be evidence of change with respect to all three categories listed.	Lesson has at least three valid changes. There is evidence of change with respect to at least two categories listed.	Lesson has less than three valid changes. There is some evidence of change with respect to some of the categories listed.	(/4)
Research-Based Practices from Foundations of Math (The Components of Number Sense, Language, Diagram Literacy, Connection to Prior Learning; Explicit Instruction)	At least four researched-based practices are addressed with specific examples.	At least three researched-based practices are addressed with specific examples.	Two or fewer researched-based practices are addressed and/or the examples are not specific.	Only one researched-based practice is addressed and/or there are no examples of the researched-based practice.	(/4)

Total

Passing Score 11/12 (/12)

Number Knowledge Test Follow-Up and Assessment with Learning Plan

Participant Name | Level 2 | Learning Task 6 and 8 | Rubric

CATEGORY	4	3	2	1	Category Score
Assessment Tools and Analysis	The recording document(s) for the Number Knowledge Test and any additional probes and checklists are included and accurately analyzed.	The recording document(s) are not included. Information from the assessments are included and accurately analyzed.	The Number Knowledge test data is not used to narrow appropriate probe choices. Actual probes or checklists are included along with an accurate analysis of those probes.	The recording documents from any of the probes are not included, but an attempt was made to analyze the data from the assessment session.	(/4)
Learning Plan (Initial Goal)	A specific and appropriate initial instructional target is outlined.	An appropriate initial instructional target is outlined.	An instructional target is outlined but is not the optimum instructional target for the assessed student.	An initial instructional target is not identified.	(/4)
Learning Plan (Improvement Goals and Objectives and Activities)	At least three specific goals and objectives are listed with at least one structured learning activity for each goal is submitted.	At least two specific goals and objectives are listed with at least one structured learning activity for each goal is submitted.	One goal and objective is listed with at least one specific structured learning activity is submitted for this goal.	Only goals or learning activities are submitted.	(/4)
Learning Plan (Monitoring) (Persons responsible for the intervention; Assessment Probes; Probe Analysis)	At least two sets of probes are included to monitor the learning progress and are accurately analyzed. The interventionist's title is listed.	At least one set of probes are included to monitor the learning progress and is accurately analyzed. The interventionist's title is listed.	One or two sets of probes are included, but the analysis is not accurate in determining the next steps in the instructional plan. The title of the interventionist may or may not be included.	There are no probes and/or analyses included. The title of the interventionist may or may not be listed.	(/4)

Passing Score 14/16 (/16)

Individual Learning Plan

Student Name_____

Assessment Result

Assessment	Results	Observations

Improvement Goals and Measures:

_____

_____

_____

_____

_____

_____

_____

Person(s) Responsible for Interventions

_____

_____

_____

_____

Rubric for Probes
Week 1

Goal	**Always 90%-100%**	**Sometime 70%-89%**	**Not Yet Below 70%**

Unit 1

REQUIRED READING

HANDOUTS

Find individuals that can attest to one of the boxes below. Have that person initial in the box. Each person can initial once per sheet. Fill the entire sheet to complete the bingo board.

Has more than two cats	Has been teacher of year	Has a child in college	Has a bird for a pet	Has two masters degrees
Has been out of the USA in the last year.	Reads the newspaper every morning	Has flown in the last year	Plays golf	Has lived in more than three U.S. states
Has blue eyes	Has taught more than 15 years	Has more than 2 grandchildren	Has a Karaoke Machine	Snow skis
Speaks two languages	Is a math teacher	Has a cell phone with a picture of their significant other.	Has more than two children	Enjoys gardening
Plays a musical instrument	Has a significant other that is also a teacher	Has taught less than three years	Has an Ipod/IPhone with them.	Was born out of the county

Math Foundations Trainer Requirements Checklist for Responsibilities

1. Completion of Level 1 requirements earns 4 CEUs.

Requirements include:

- **100% attendance, participation in all 5 days.**
- **Study readings & respond to discussion questions assigned for Level 1 training.**
- **Completion of all learning tasks assigned for Level 1 training.**
- **Successful completion of submitted homework acknowledged by trainers.**

2. Completion of Level 2 requirements earns 3 CEUs.

Requirements include:

- **100% attendance, participation in all 5 days (one of the two training weeks must be training sponsored by DPI).**
- **Completion of all assignments and homework acknowledged by trainers.**

3. Assessment of conceptual math knowledge.

Requirements to become a Math Foundations Trainer include:

- **Aspiring trainers are responsible for documenting completion of Levels 1, 2 and all other trainer requirements. Documentation examples include: e-mails, signatures.**
- **Schedule math trainer assessment with DPI within 1 year of completion of Level 2 training by contacting the DPI Math SIP Coordinator.**
- **Pass the trainer assessment with 80% accuracy (PRIOR to scheduling any training in LEA).**
- **Send training dates to DPI six weeks PRIOR to training (to allow time for scheduling of observations).**
- **Team train during first training with a satisfactory evaluation by a master trainer.**
- **Complete three required observations. Two of these observations are scheduled during training days 2, 3, or 4.**
- **The final observation is submitted as a video tape with self-reflection completed. Directions and forms for the self-reflection will be provided when trainee is at this point.**
- **Submit documentation of all completed requirements to the LEA SIP Coordinator and DPI Math Sip Coordinator to be able to:**
- **Attend and receive certification at Training of Trainers event.**
- **Celebrate this accomplishment!**

*****Principal and EC Director Approval (signature) are required to become a Math Foundations Trainer.**

POWERPOINTS

Public Schools of North Carolina

NCSIP: Foundations of Mathematics Course

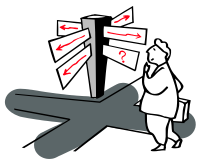
Research to Practice

Public Schools of North Carolina

Unit 1
Foundations of Mathematics: Research to Practice

Texts:
Liping Ma Knowing and Teaching Elementary Mathematics
James Royer Ed. Mathematical Cognition
(Recommended)

Note to Participants:



Public Schools of North Carolina

Public Schools of North Carolina

Unit 1: Foundations of Mathematics

- Purpose and Overview of Course
- Selection of Mathematics Programs
- Components of Effective Implementation

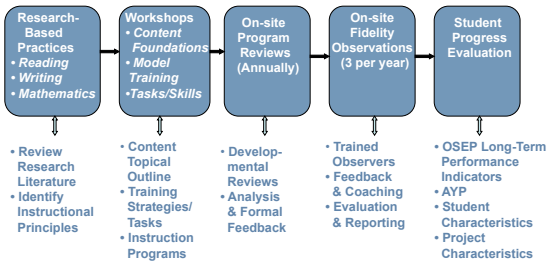
NCSIP: Purpose

North Carolina State Improvement Project (NCSIP) works to significantly improve the performance and success of students with disabilities in North Carolina.

Public Schools of North Carolina

Public Schools of North Carolina

NCSIP Personnel Development Process



Course Purpose

- Increase understanding of the scientific research-based instructional principles
- Increase knowledge and skills for implementation of research-proven teaching strategies for students with persistent Mathematical problems.

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Why Does the Course Matter ?

"I am indebted to my father for living, but to my teacher for living well."

Alexander the Great

Public Schools of North Carolina

Why Does The Course Matter?

"Effective teachers are the only absolutely essential element for an effective school."

Allington & Cunningham, 1996

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Why Does The Course Matter?

"Research has borne out that the key factor in students' achievement is the quality of teaching... Teachers are central to the process of education, assessing student's progress, selecting and using a variety of approaches and materials, and organizing for instruction."

Braunger & Lewis, 1999

Course Goals

Course Goals

- To develop participant understanding of basic principles of effective teaching and how they apply to instruction in the math content area
- To increase participant understanding of the importance of language with mathematics instruction for all children

Course Goals (continued)

- To increase participant understanding of math difficulties and how to help struggling math students
- To provide opportunity and develop skills of participant to review, discuss, and make sound judgments about research, instructional practices, and materials

Course Topics

- Overview of Course and Research
- Demystifying Math
- Components of Number Sense
- Quantity/Magnitude & Numeration
- Equality, Base Ten, & Form of a Number
- Proportional Reasoning & Algebraic and Geometric Thinking
- Assessment
- Connections and Reflections

Options for Participation

Requirements For Level 1 Foundation Training

Completion of Level 1 requirements earns 4 CEUs.

Requirements include :

- Research agreement to use data from pre and post tests.
- 100% attendance & participation in all 5 days.
- Study readings & be prepared to respond to Discussion Questions appointed for Level 1 training.
- Complete all Learning Tasks appointed for Level 1 training.
- Participate in group tasks.

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Requirements For Level 2 Foundation Training

Completion of all Level 2 requirements earns 3 CEUs & qualifies the participant to enter training to become a Foundations of Math trainer.

Requirements include :

- All of the requirements of Level I.
- Complete Discussion Questions and Learning Tasks appointed for Level 2 training.
- Complete the entire training twice. One training must be a state level training.
- Demonstrate 80% accuracy on the trainer assessment.
- Team train in your first training with a satisfactory evaluation by a master trainer.
- Observations on training days 2, 3, and 4 will be done. Final observation will be a video tape with reflection submission.

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Intervention

- We now know a lot about **WHAT** to do to educate students
- We can improve education for students – on purpose!

Fixsen (2010)

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CAUTION!

Students cannot benefit from interventions they do not experience

Teachers and staff have to change if students are to benefit

Fixsen (2010)

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Unit 1: Foundations of Mathematics

- Purpose and Overview of Course
- Selection of Mathematics Programs
- Components of Effective Implementation

Questions To Answer About Mathematics Programs

- Is it scientifically research-based?
- Does it contain multisensory strategies?
- Does it include systematic, explicit and direct instruction?
- Does it give attention to understanding fundamental operations with number?
- Does it provide teacher support for working with students at different levels?
- Does it include organized on-going assessments?

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Other Questions To Ask About Mathematics Programs

- Is training and/or mentorship required for the program?
- Is there a cost for student and teacher materials and/or the training?
- Is there software and on-line support?
- Does the program contain placement tests?
- Are there benchmark assessments to use at various points in the program?

Examples of Research-Based Math Programs used in NC:

Transitional Mathematics
Number Worlds
Math Expressions
Voyages

Note: This is not an exhaustive list.

Transitional Math

- Fewer topics in more depth
- Provides visual representations to help conceptualize the mathematics
- Meets individual student needs
- Provides a logical sequence, ample practice, and an appropriate pace
- Aligns with National Council of Teachers of Mathematics (NCTM) Standards
- Provides a balance between procedural knowledge and conceptual understanding

John Woodward ,
University of Puget Sound

Number Worlds

- Developed by Sharon Griffin
- Teachers specific math concepts and skills that are foundational for later mathematical learning
- Focuses on development of Number Lines and connections across concepts in early grades.
  - Source: Number Worlds, Griffin

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Number Worlds

Number Worlds Home

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Web Resources

- http://ncsip.org (SIP site)
- www.nrcld.org (National Research Center for Learning Disabilities)
- http://iris.peabody.vanderbilt.edu/ (Research to Practice)
- http://www.whatworks.ed.gov (What Works)

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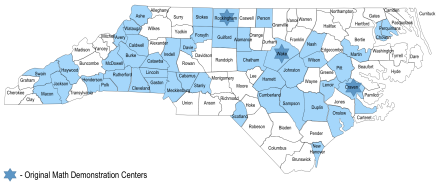
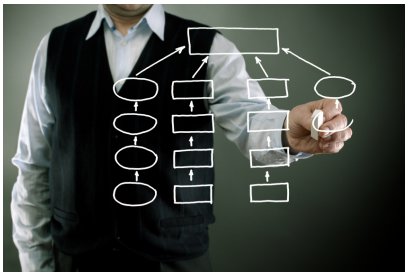
North Carolina
Math State Improvement Project
2011-2012



- Original Math Demonstration Centers

Additional LEAs involved in the Math SIP:
Asheboro City, Asheville City, and Roanoke Rapids

Got a Plan ?



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What is the Purpose of the Five Year Plan?

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"Reform by the Book"

1. Why have curriculum materials played an uneven role in teacher practice?
2. What are the influences that teachers have in enacting the curriculum?
3. What contributions might curriculum materials make in enacting the curriculum?
4. What are some considerations with regard to curriculum materials?

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Unit 1: Foundations of Mathematics

- Purpose and Overview of Course
- Selection of Mathematics Programs
- Components of Effective Implementation

Developing An Implementation Plan

How do we know it works?

- Frequent assessment of students
- Assessment drives instruction
- Formal review process of student progress and program effectiveness
- Strong leadership and commitment of all involved

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To Be Effective, Instruction For Students With Reading Difficulties, Must Be…

"more intensive, more relentless, more precisely delivered, more highly structured and direct, and more carefully monitored for procedural fidelity."

Ken Kavale, 1996

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To Be Effective, You Must:

- Know your stuff,
- Know who you're stuffing,
- Know why you're stuffing,
- Stuff every minute of every lesson.

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The North Carolina State Improvement Project THANKS YOU for your time and support.

Questions:
919-843-5037
javeritt@northcarolina.edu
www.ncsip.org

Unit 2

REQUIRED READING

HANDOUTS

Benchmarks

The benchmarks set forth by the Panel should help to guide classroom curricula, mathematics instruction, textbook development and state assessments that will lead to proficiency with whole numbers and fractions, and competence with certain aspects of geometry and measurement.

Grade Level (by the end of)	Mathematics Learning Goal
Grade 3	Proficiency with addition and subtraction of whole numbers.
Grade 4	Ability to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.
Grade 5	Proficiency with multiplication and division of whole numbers. Proficiency with comparing fractions and decimals and common percents, and with the addition and subtraction of fractions and decimals. Ability to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
Grade 6	Proficiency with multiplication and division of fractions and decimals. Proficiency with all operations involving positive and negative integers. Ability to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.
Grade 7	Proficiency with all operations involving positive and negative fractions. Ability to solve problems involving percent, ratio, and rate and extend this work to proportionality. Familiarity with the relationship between similar triangles and the concept of the slope of a line.

Foundations for Success

Findings and Recommendations From the National Mathematics Advisory Panel

The National Mathematics Advisory Panel
http://www.ed.gov/MathPanel

For electronic copies of the Math Panel's final report and the report of the task groups and subcommittees, please visit www.ed.gov/mathpanel or order copies at http://edpubs.ed.gov/, by e-mail at edpubs@inet.ed.gov and by phone at 1-877-433-7827 (TDD/TTY 1-877-576-7734).

About the National Mathematics Advisory Panel

For students to compete in the 21st-century global economy, knowledge of and proficiency in mathematics are critical. Today's high school graduates need to have solid mathematics skills—whether they are headed to college or to the workforce. To help ensure our nation's future competitiveness and economic viability, President George W. Bush created the National Mathematics Advisory Panel (National Math Panel) in April 2006. The Panel was charged with reviewing the best available scientific evidence and making recommendations on improving mathematics education with a focus on readiness for and success in algebra.

The National Math Panel's final report, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, was issued on March 13, 2008. It contains 45 findings and recommendations on numerous topics, including curricular content, learning processes, instructional practices and materials, teachers, assessments and future research priorities. Highlights of the findings and recommendations are briefly summarized below. For a complete discussion of these topics, the public may read the final report as well as the reports of the task groups and subcommittees by visiting www.ed.gov/mathpanel or by ordering copies at http://edpubs.ed.gov/, by e-mail at edpubs@inet.ed.gov and by phone at 1-877-433-7827 (TDD/TTY 1-877-576-7734).

Core Principles of Math Instruction

The areas to be studied in mathematics from pre-kindergarten through eighth grade should be streamlined and a well-defined set of the most critical topics should be emphasized in the early grades. Any approach that revisits topics year after year without bringing them to closure should be avoided. Other important findings and recommendations for mathematics instruction are:

- Critical Foundations. Proficiency with whole numbers, fractions, and certain aspects of geometry and measurement are the critical foundations of algebra.
- Fractions. Knowledge of fractions is the most important foundational skill not currently developed among American students.
- Conceptual understanding, computational and procedural fluency, and problem-solving skills are equally important and mutually reinforce each other. Debates regarding the relative importance of each of these components of mathematics are misguided.
- Automaticity. Students should develop immediate recall of arithmetic facts to free the "working memory" for solving more complex problems.
- Major Topics of School Algebra include Symbols and Expressions; Linear Equations; Quadratic Equations; Functions; Algebra of Polynomials; and Combinatorics and Finite Probability. More students should be prepared for and offered an authentic algebra course in Grade 8.

Other Key Findings

- Effort matters. A focus on the importance of effort will improve outcomes. If children believe that their efforts to learn make them "smarter," they show greater persistence in mathematics learning and their performance improves.
- Most children develop considerable knowledge of mathematics before they begin kindergarten. Children from families with low incomes and low levels of parental education and those of single parents often have less mathematical knowledge when they begin school than do children from more advantaged backgrounds. This tends to hinder their learning for years to come. There are promising interventions to improve the mathematical knowledge of these young children before they enter kindergarten.
- Teachers' mathematical knowledge is important for students' achievement. The preparation of elementary and middle school teachers in mathematics should be strengthened. Teachers cannot be expected to teach what they do not know.
- Teachers who consistently produce significant gains in students' mathematics achievement should be recognized. When students are taught by a series of effective teachers, the positive outcomes on their mathematics learning are compounded.
- Teachers' regular use of formative assessment improves their students' learning, especially if teachers have additional guidance on using the assessment to individualize instruction.
- Explicit instruction with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computation.
- More research is needed. The nation must continue to build the capacity for more rigorous research in mathematics education to more effectively inform policy and practice.

The National Mathematics Advisory Panel

24 expert panelists, including a number of leading mathematicians, cognitive psychologists and mathematics educators, reviewed more than 16,000 research studies before preparing a final report containing policy advice on how to improve mathematics achievement for all students in the United States.

Next Steps

The findings and recommendations of the Panel are a first step in the process toward fixing the delivery system in mathematics education in this country. The United States has genuine opportunities to improve mathematics education, but these improvements can be realized only if educators, policymakers, researchers and parents all work together to ensure that our students master the mathematical foundations that are so critical for achievement in algebra and beyond.

POWERPOINTS

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Unit 2

What the Research Says----

According to the 2011 National Assessment of Educational Progress (NAEP) only 40% of fourth-grade students and only 35% of eighth-grade students scored at the proficient level in mathematics. (NC: 45%;37%)

Feedback?

National Center for Educational Statistics, 2009

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- Schmidt, Houang, and Cogan (2002) reported that by the end of high school, US students performed near the bottom of the international distribution in the Third International Mathematics and Science Study (TIMSS).
- 2007 Average Mathematics Scores for US student higher than TIMSS averages

TIMSS 2007 Country Comparison for Mathematics (Grades 4 and 8)

Feedback?

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The National Council for Teachers of Mathematics (NCTM)

highlights the need for a well designed curriculum and quality teacher preparation.

Federal Recognition of Lack of Research Studies

- IDEIA 2004 (Individuals with Disabilities Education Improvement Act) and NCLB (No Child Left Behind) clearly define a high standard for research-based reading practices
- IDEIA 2004 did not clearly define a high standard for research-based math practices, because we did not have the same research for math as we did for reading.

National Mathematics Panel Report 2008 What do students need for success in Algebra?

•Proficiency with whole numbers, fractions and certain aspects of geometry and measurement are the critical foundations of algebra

•Explicit instruction for students with disabilities shows positive effects.

•Students need both explicit instruction and conceptual development to succeed in math.

International Research

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TIMSS

from Improving Mathematics Instruction (Ed Leadership 2/2004)

- 1995 Video Study
  - Japan, Germany, US
  - Teaching Style Implicated
- 1999 Video Study
  - US, Japan, Netherlands, Hong Kong, Australia, Czech Rep.
  - Implementation Implicated

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High Achieving Countries
MAKE CONNECTIONS

U.S.
TEACHES PROCEDURES

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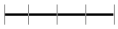
Structures and Connections

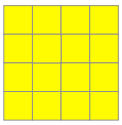
What is 4^2 ?

Procedure versus Structure/Connections

Make a square out of your 4 unit linear side

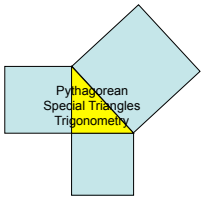


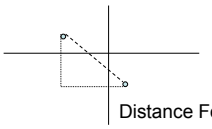
Exponents and CONNECTIONS



Square Roots!
$\sqrt{16} = 4$
The length of one side!

Geometry and Measurement

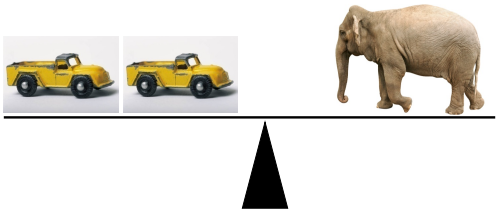




Distance Formula

$$1 = 1^3$$

Are these the same?



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Concept Development & Connections

"Research has shown a mutually beneficial pairing of procedures and concepts."

--David Chard

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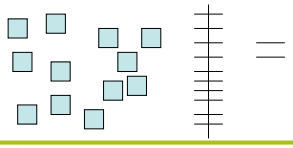
C-R-A Cecil Mercer

The student moves through stages.

The teacher has the responsibility to explicitly and directly instruct students through these stages.

Make connections for the students!

Concrete → Representational → Abstract



$7 + 5 = 12$

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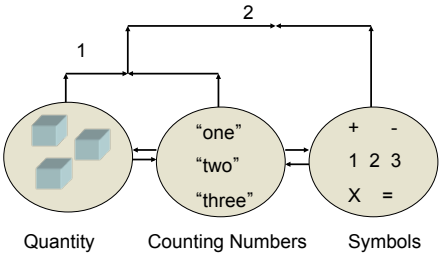
Concrete Reality

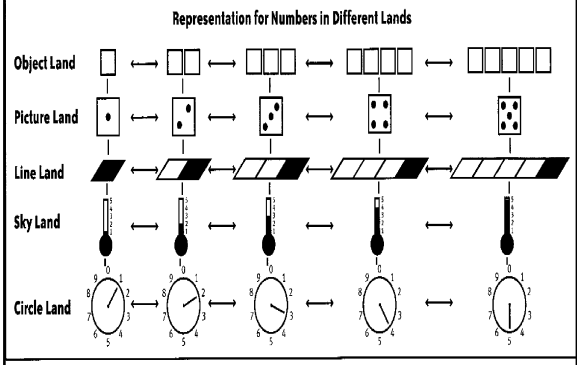
8 - 5 = 8

7 - 4 = 7

Sharon Griffin
Core Image of Mathematics

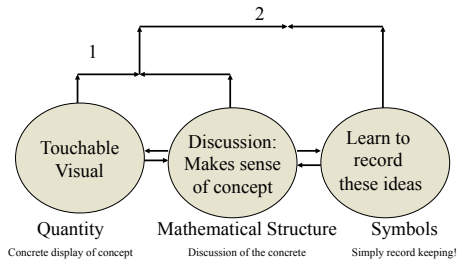


V. Faulkner and DPI Task Force adapted from Griffin



Griffin

Prototype for lesson construction



V. Faulkner and DPI Task Force adapted from Griffin

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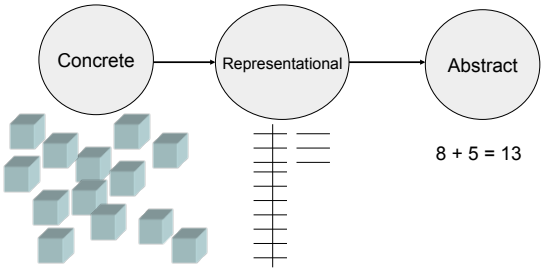
CRA instructional model



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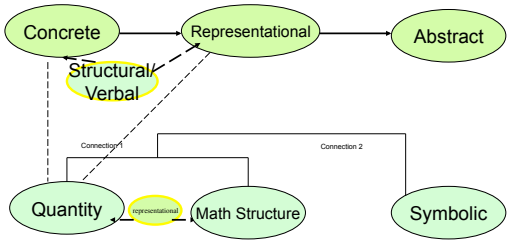
Synthesizing CRA & Griffin Model

CRA (Instructional Model)



Griffin (Cognitive Development Model)

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We teach *digitally* but we ALL have **analog** brains!

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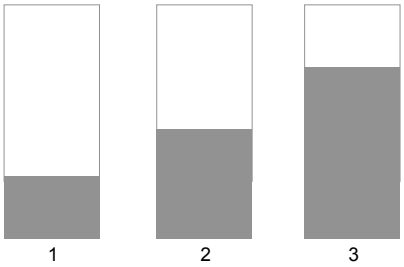
The Accumulator Model: Our Analog Brain



See Stanislas Dehaene 1999. *The Number Sense*

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Language, Reading and Mathematics

Connections and Disconnections

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Connections and Disconnections

- Oral Language
- Reading and Writing
- Mathematics

Characteristics of Students with Learning Difficulties in Mathematics

- Slow or inaccurate retrieval of basic arithmetic facts
- Impulsivity
- Problems forming mental representations of mathematic concepts (number line, visual means to represent subtraction as a change process) (Geary 2004)
- Weak ability to access numerical meaning from mathematical symbols (i.e. poorly developed number sense) (Gersten and Chard 1999 and Noel 2006)
- Problems keeping information in working memory. (Passolunghi and Siegel 2004; Swanson and Beebe-Frankenberger 2004)

(adapted from Gersten and Clarke. NCTM 2007)

Gellman and Gallistel's (1978) Counting Principles

- 1-1 Correspondence
- Stable Order
- Cardinality
- Abstraction
- Order-Irrelevance

Geary and Hoard, Learning Disabilities in Basic Mathematics from Mathematical Cognition, Royer, Ed.

Briars and Siegler (1984) Unessential features of counting

- Standard Direction
- Adjacency
- Pointing
- Start at an End

Geary and Hoard, Learning Disabilities in Basic Mathematics from Mathematical Cognition, Royer, Ed.

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Students who are Identified with a Math Disability

Error: Double Counts

Working memory is a key factor!

Geary and Hoard, Learning Disabilities in Basic Mathematics from Mathematical Cognition, Royer, Ed.

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SPED Research

- Concrete-Representational-Abstract
- Meta-cognitive support
- Routine and Structure
- Teach the BIG IDEAS
- Direct Instruction: Systematic, Explicit, Cumulative and Multi-Sensory

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Considerations Regarding Instruction and Interventions

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Targeted Instruction is

- Explicit
  - Focus on making connections
  - Explanation of concepts
- Systematic
  - Teaches skills in their naturally acquired order
- Multi-sensory
- Cumulative
  - Connecting to prior knowledge and learning
- Direct
  - Small group based on targeted skills
  - Progress monitored

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Instructional Design Concepts

- Sequence of Skills and Concepts
- Knowledge of Pre-skills
- Example Selection
- Practice and Review

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Sequencing Guidelines

- Pre-skills of the mathematical concept or skill are taught before a strategy

  4008
  - 9

- Information presented in concrete→representational→abstract sequence

Preskills

Component skills of any strategy should always be taught before the strategy itself.

What is 45% of 80?

- What are the pre-skills?

Selecting Examples

- First only use problems with the new material in practice exercises.
  - Few programs provide a sufficient number of examples for new material – and do not emphasize mathematical connections.
  - The programs rarely present a discussion of examples versus non-examples to help frame students' thinking.

- Secondly, we integrate problems with both old and new material.
  - The purpose of this:
    - is to allow discrimination between using the new strategy and not using the strategy.
    - is to allow practice of previously learned skills (cumulative review).
      - Research suggests a strong relationship between retention and practice & review (Stein, 2006)

What are the components of a practice set?

- Mastery level of <u>understanding</u>
  - What if you made a practice of sending home two or three different sets of HW? (93-100% accuracy)
  - This practice can be used to build automaticity.
  - Homework should not be seen as an opportunity for student to learn a skill, simply to correctly practice that skill.

Practice and Review

Mastery is reached when a student is able to work problems with automaticity and fluency (fact learning).

Provide systematic review of previously learned skills.

You can only spiral back to something that you know and have mastered.

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Anxiety, Performance and Instructional Considerations

Testing
Grading opportunities
Self-concept

"Understanding Math Anxiety"

1. What implications does the article make with respect to math anxiety?
2. What can we do as instructors to ease the trauma of math anxiety?

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Some words about "Key Words"

They don't work…

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We tell them—more means add

Erin has 46 comic books. She has 18 more comic books than Jason has. How many comic books does Jason have.

But is our answer really 64 which is 46 + 18?

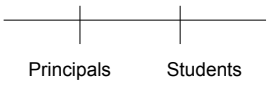
Sense-Making

- We need to notice if we are making sense of the math for our students, or if our discussion of the math contributes to ---

"The suspension of sense-making…"
Schoenfeld (1991)

Mayer

- Translating
  - Relational sentences and reversal errors
    - There are 6 students for every principal



Mathematically Authentic Instruction

- Research indicates that teachers who have a more robust understanding of the math present math in a more accurate and effective manner.
- Deeper understanding on the part of teachers positively affects student performance.

Deborah Ball and Liping Ma

Public Schools of North Carolina

Public Schools of North Carolina

Differentiation

Sameation

"Teaching Number Sense"

- **What is Number Sense and how has the definition changed through time?**
- **Why is the distinction between these two definitions important in the opportunities that students are given to learn mathematical concepts?**
- **What signals the acquisition of number sense?**
- **What are the three worlds that comprise mathematics according to Griffin?**
- **What is the primary difference in conceptual understanding at age seven versus age nine?**
- **What instructional principles should we consider to teach Number Sense?**

Sharon Griffin

Public Schools of North Carolina

Assignments from Units 1 and 2

Due Day 2

Readings for Units 3 and 4:

- Assigned chapter from *Knowing and Teaching Elementary Mathematics* (Ma)
- "Arithmetic in American Mathematics Education" (Ma)
- "Magical Hopes" (Ball)
- "Components of Number Sense: An Instructional Model for Teachers" (Faulkner)

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Assignments for Liping Ma

- Chapter 1-Subtraction
- Chapter 2- Multiplication
- Chapter 3- Division of Fractions
- Chapter 4- Area and Perimeter
- If you have already read one of these chapters, choose a different one to read. (Level 2 Participants)

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Be prepared to discuss in Expert Groups

- The approach of the U.S. Teachers to the topic
- The approach of the Chinese Teachers to the topic
- Biggest pedagogical shift you may need to take regarding this topic and/or the most interesting idea you learned.

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References

- Chard, D. (2004) Council for Exceptional Children Conference. New Orleans, LA
- Geary and Hoard, Learning Disabilities in Basic Mathematics
  from Mathematical Cognition, Royer, Ed.
- Griffin, S Number Worlds (2004)
- National Center for Educational Statistics, 2007
- Schmidt, Houang, and Cogan (2002)
- TIMSS, Improving Mathematics Instruction (Ed Leadership 2/2004)
- Dehaene, Stnailsas, 1999 The Number Sense

Unit 3

REQUIRED READING

HANDOUTS

Cover Up

You need:

your fraction kit
a fraction die with faces marked 1*2, 1\4, 1\8,1\8, 1\16, 1\16,*
a partner

Rules

1. Take turns rolling the fraction die.

2. On your turn, the fraction that comes up on the die tells what size piece to place on the whole strip.

3. Check with your partner to be sure he or she agrees with what you did.

4. After finishing your turn, say "Done" and pass the die to your partner.

5. The first player to cover his or her whole strip exactly wins. If you need only a small piece—1/8 or 1/16, for example—and you roll a larger fraction—1/2 or 1/4, example—you can't play. You must roll a fraction smaller than or exactly what you need.

 From *The Fraction Kit Guide, Grades 4-6* by Marilyn Burns. © 2003 Math Solutions Publications

Uncover, Version 1

You need:

your fraction kit

a fraction die with faces marked 1*2, 1\4, 1\8, 1\8, 1\16, 1\16*

a partner

Rules

1. Each player covers his or her whole strip with the two \ pieces.
2. Take turns rolling the fraction die.
3. On your turn, take one of three options:

   - remove a piece (only if you have a piece the size indicated by the fraction facing up on the die);
   - exchange any of the pieces on your whole strip for equivalent pieces;
   - do nothing

4. Check with your partner to be sure he or she agrees with what you did.
5. After finishing your turn, say "Done" and pass the die to your partner.
6. The first player who removes all pieces from the whole strip wins.

NOTE 1: You may not remove a piece and exchange on the same turn; you can do only one or the other.

NOTE 2: You have to go out exactly. That means if you have only one piece left and roll a fraction that's larger, you may not remove the piece.

From *The Fraction Kit Guide, Grades 4-6* by Marilyn Bums. © 2003 Math Solutions Publications

POWERPOINTS

Public Schools of North Carolina

Teacher Understanding Unit 3

Deborah Ball

Defining Issue in Implementation

...is the teacher's **own** understanding of Mathematics.

--Liping Ma

Understanding and Instruction

We can only instruct our students as well as we understand the mathematics:

The better we understand the math, the better decisions we will make regarding what the student needs to achieve and how to instruct the student!

Knowing and Teaching Elementary Mathematics (Liping Ma)

- Compared and contrasted the pedagogy of Chinese and American teachers
- Found that American teachers were much weaker in content and conceptual knowledge
- Found American teachers teach procedurally rather than being driven by the logic of the mathematics (implementation)
- Ma presented information through teacher responses to elementary math questions

Expert Groups

- Chapters 1, 2, 3 and 4
  - Approach of the American teachers
  - Approach of the Chinese teachers
  - Largest pedagogical shift in thinking

Problem #1 Subtraction

$$\begin{array}{r} 72 \\ \underline{-15} \end{array}$$

How would you approach this type of problem if you were teaching second grade?

Problem #1
Subtraction with Regrouping

American Teachers—Procedural approach

- The "pedagogic insight "of the teachers
  - Once the student can take a ten from the tens place and turn it into 10 ones, then they can address the problem correctly. Problem solved.
  - Manipulatives suggested to explain this step only.
  - Manipulatives sometimes used in a way that they did not actually demonstrate process of regrouping.

Problem #1
Subtraction with Regrouping

Chinese Teachers—Decomposing and Composing a Higher Value Unit

- Pedagogical insight: implemented the package of critical information embedded in subtraction
- Saw this problem as connected to addition through composing and decomposing units
- Demonstrated multiple ways of regrouping
- Found the opportunity to explore the basics of our base ten system

Math "Facts" vs. Subtraction within 20

- U.S. deals with problems like 12-5, 15-7, etc., as FACTS to be memorized.
- It does help to have these memorized, however, PEDAGOGICALLY, there's more to it.
- Chinese teach these "facts within 20" as the entry point for understanding our number system (develop "number sense", emphasize base ten system)

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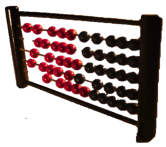
15
- 8
7

Equal Exchange, Base Ten and Form of a Value



Develop and ingrain the idea of equal exchange and the base ten system

- 1's cubes and 10's rods to enforce rate of exchange
- Solve "problems within 20" emphasizing this base ten system and rate of exchange

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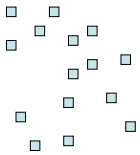
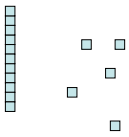
Pop Cubes and Tens "Composing and Decomposing"

Compose tens



Decompose tens



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Language Tips:
Subtraction within 20

- Standard Form and Ones Form
- Equal refers to value
- A ten rod is not the same as ten ones
- Chinese language: “1 ten 2 ones”
- Mathematicians evaluate the form
  (e.g., “Is this the form I want my value in?”)

Develop an Understanding of Base 10 and Equal Exchange

16-4	14-7
12-8	17-3
13-5	19-9
18-9	15-6

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Know what you are teaching!

- 15-8 “Number Fact” (Automaticity)
- 15-8 Unlocking the Number System

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Structures of Subtraction

The Classic "Take away" (How many are left?)
You've got some amount and "take away" from that amount. How many are left?

Comparison (Difference between? Who has more?)
You compare to see who has more or less?

Deficit/Missing amount (What's missing?)
You need some more to get where you want to be. What is the missing amount?

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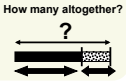
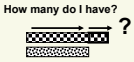
Structures of Addition

Join and Part-Part Whole
- There is something, and you get more of it?
- There are two kinds, how many all together?



Start Unknown
Some are given away, some are left, how many were there to start?

Compare--total unknown
I know one amount and I have some amount more than that. How many do I have?



Adapted from Carpenter, Fennema, Franke, Levi and Empson, 1999 p. 12 in Adding it Up, NRC 2001.

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Problem #2 Multiplication

123
645
615
492
738
1845

What's happening here?
How would you correct this mistake?

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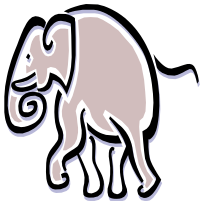
Problem #2 Multiplication

American Teachers Approach

- Lining Up correctly
- American teachers saw this as a problem of alignment and thus addressed with systems or "tricks" for alignment
- Teacher's understanding reflects the way they were taught multiplication



Problem #2 Multiplication

Chinese Teachers Approach:

Elaborating the Concept of Place Value and Powers of 10

- Reinforce concept of Base 10 system
- Develop concept of Distributive Property
- Place Value a logical system (not so much a "place")
- Developing foundation and connections for higher thinking in mathematics

1
~~2~~
23
x 47
161
920
1081

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```
   23
 x 47
   21
  140
  120
  800
 1081
```

Problem #2
Multiplication

Develop understanding of the numbers:
123= (1) x 100 + (2) x 10 + (3) x 1
645= (6) x 100 + (4) x 10 + (5) x 1

Also we could ask:
What does 123 x 5 represent?
What does 123 x 4 represent?
What does 123 x 6 represent?

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Multiplication and Properties

- Distributive Property
- Reinforce the concept of 10s
- 12 • 5 = (10 + 2) • 5
- 60 = 50 + 10
- 60 = 60
- Reinforce willingness to “play” with numbers
- 12 • 5 = (6 + 6) • 5
- 60 = 30 + 30
- 60 = 60

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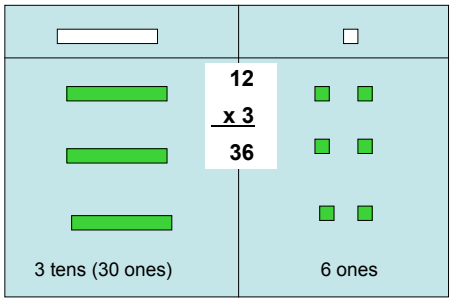
Concrete Multiplication



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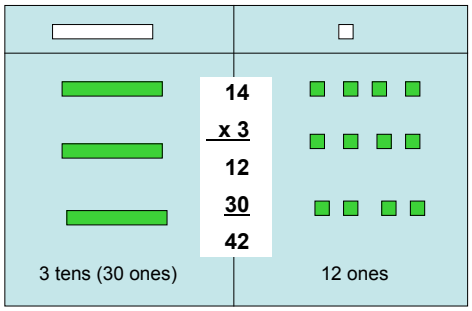
Concrete Multiplication with Regrouping



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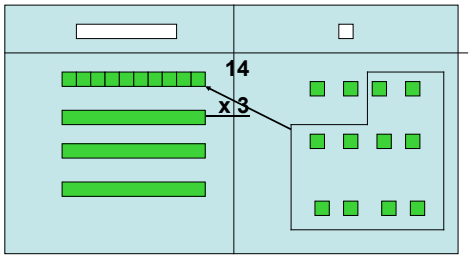
Develop the Distributive Property and Reinforce Base Ten



14x 3 = 3(10+4)= 30 + 12
(12 ones = 1 ten and 2 ones)= 10 + 2
30 + 10 + 2 = 42

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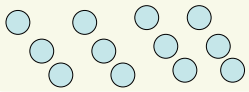
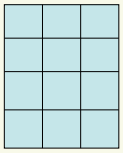
STRUCTURE:

3 Types of Multiplication: 4 x 3

Repeated Addition



Array/
row-column



Counting Principle

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Know what you are teaching?

- Multiplication --Place value as a place
  Multiplication as an algorithm
- Multiplication --Unlocking the System

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Problem #3 Division of Fractions

1 ¾ divided by ½

Give a Story Problem to show what is happening with this expression.

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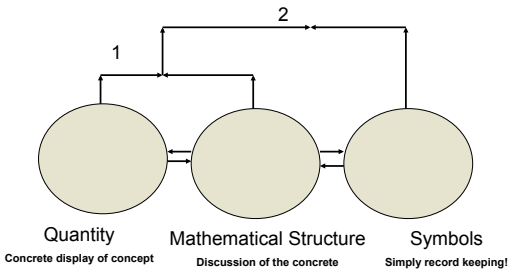
Division of Fractions Lesson Construction



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V. Faulkner and NCDPI Task Force adapted from Griffin

Problem #3 Division of Fractions

1 ¾ divided by ½

Give a Story Problem to show what is happening with this expression.

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Division of Fractions

American teachers' approach

- Flip and multiply
- Answers don't match
- Confused division by two with division by one-half

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Division of Fractions

Chinese teacher's approach

- Gave a mathematically accurate story problem
- Explained the mathematics behind the operation
- Gave multiple mathematical constructs for division of fractions

Division of Fractions

- What does it mean to divide by a fraction?

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Division as Repeated Subtraction

Bean Party!

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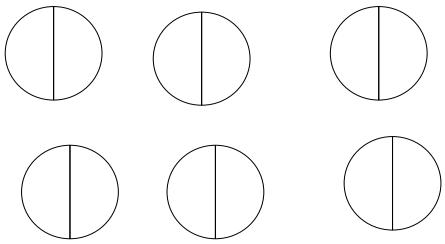
For each Cup of Beans—
I get two portions if I divide by 1/2



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What if I divide by 1/3?

- How would my diagram look?
- How many portions would I have?
- Why are my number of portions getting larger?
- How does this idea of quantity tie into the math structure of proportional reasoning and repeated subtraction?

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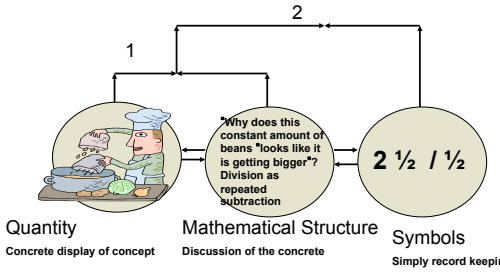
Division of Fractions: Bean Party!
Division as Repeated Subtraction



V. Faulkner and DPI Task Force adapted from Griffin

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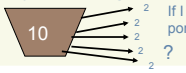
Division Structures

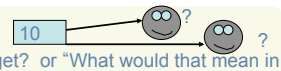
Measurement/Repeated Subtraction

"How many 2s can I get out of 10?"



If I have 10 cups of beans and I give out 2 cup portions, how many servings will that provide?

Partitive/Unitizing/Fair Shares



"How many would one person get? or "What would that mean in relation to 1?"

If 2 people find $10 how much will each person get ?

Product/Factor

"If I have an area of 10 and one side is 2, how long is the other side?"

Problem #3 Division of Fractions

Why DO we "Flip and Multiply" anyway???

Eloquent Explanation: Remember. $\frac{N}{N} = 1$

We can apply this to our problem by what the Chinese Teachers call "maintaining the value of a quotient"

$$\frac{10}{2} = \frac{10}{2} \cdot \left(\frac{5}{5}\right)$$

Teaching Operations on Fractions

"Invert and Multiply"

vs.

Developing the concept of the procedure

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More with Fractions

Marilyn Burn's Fraction Kit

Fractions and Quotients

Picture Form	Fraction Form	Division Form	Decimal Form
	_____	_____	_____
	_____	_____	_____
	_____	_____	_____
	_____	_____	_____
	_____	_____	_____

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Implementation

- Teacher's discussion of the Mathematical Structure is critical.
- Deborah Ball has found that teacher knowledge affects student growth.

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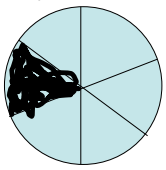
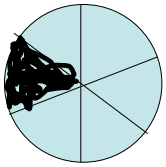
Manipulatives and "Magical Hopes"

Deborah Ball

How do you explain this to a student?



1/6 + 1/6 = 2/6



1/6 + 1/6 = 2/12

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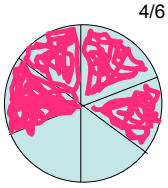
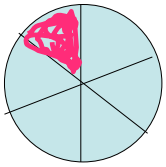
Game from SRA Real Math

I roll 5/6

What should I do with my 5/6ths?

4/6 + 1/6 = 5/6





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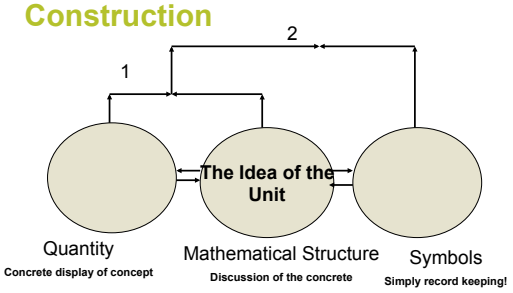
Prototype for Lesson Construction



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V. Faulkner and NCDPI Task Force adapted from Griffin

"Magical Hopes"

1. How is it that Sean thought 6 could be both odd and even?
2. Why were manipulatives not the answer for Sean?
3. What assumption do we make as teachers about the connections that students make with the mathematics?
4. How can we help students see the relationship between 4/8 and 4/4 and 3/3 and 5/5?
5. What do we need to keep in mind as we use manipulatives as a mathematical tool in developing conceptual understanding?

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Problem #4 Area and Perimeter

Imagine that one of your students comes to class very excited. She explains that she has discovered that as the perimeter of a closed figure increases the area also increases.
She shows you a picture to explain what she is doing.

P = 16cm A = 16 cm^2 P= 24cm A = 32cm^2

How would you respond to this student?

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Area and Perimeter

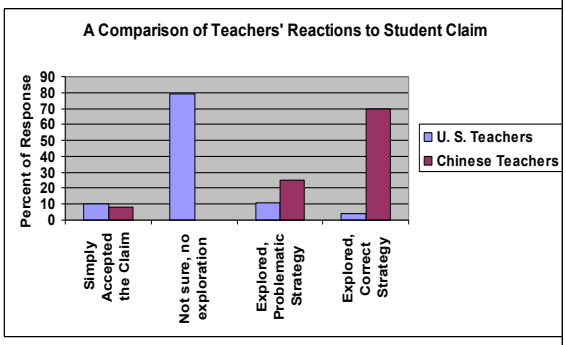
U.S. Teacher's Approach

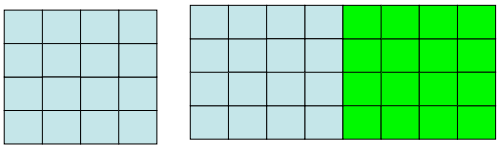
- Praised student for answer
- Looked it up in a book

Area and Perimeter

Chinese Teacher's Approach

- Understood student's misunderstanding
- Supported student in a deeper understanding of area and perimeter





Perimeter = 16 units
Area = 16 square units

Perimeter = 24 units
Area = 32 square units

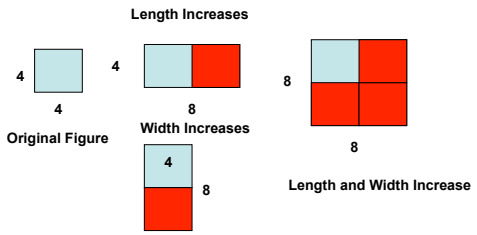
A Condition in which Student's Claim is supported

Clarifying the Conditions

- Three Cases
  - The length or width is increased
  - The length and width are increased
  - Either the length or width increases while the other dimension decreases

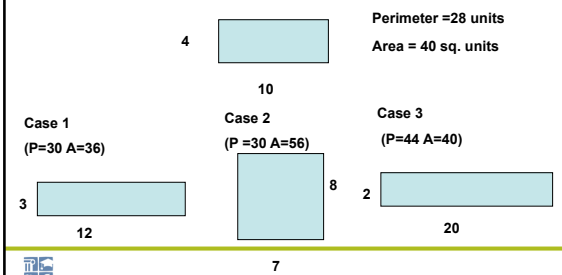
Explaining the Conditions

- In the first two conditions, an area is added.



Explaining the Conditions

- In the third condition, the area can decrease, increase or stay the same.



Solution

When an increase in the perimeter is caused by the increase in either or both the length and the width of a rectangle, the area of the figure will increase accordingly.

When an increase in the perimeter is caused by an increase in one dimension (length or width) followed by a decrease in the other dimension, the area of the figure may or may not increase.

Distributive Property

Area = LW

Area = (L + 4) W (Length is increased by 4.)

Area = LW + 4W

The additional area is 4W.

Key Factors in the Difference

Knowledge of Topic

Mathematical Attitude

- Significant in the solution of the Area and Perimeter Problem

In order to foster exploration by our students, we must embrace mathematical ways and approach new situations independently.

References

Royer, J. M. (Ed.) (2003). Mathematical Cognition. Greenwich, CT.: Infoage Publishing.

Ma, Liping (1999) Knowing and Teaching Elementary Mathematics. Edison, NJ, Lawrence Erlbaum Associates.

Ball, Deborah (1992) "Magical Hopes: Manipulatives and the Reform of Math Education", American Educator, Summer 1992

Ball's Website: http://wwwpersonal.umich.edu/~dball

Unit 4

REQUIRED READING

HANDOUTS

Trashketball Stats

Group Name _____

Group Members: 1._____,(2)_____,(3) _____(4) _____

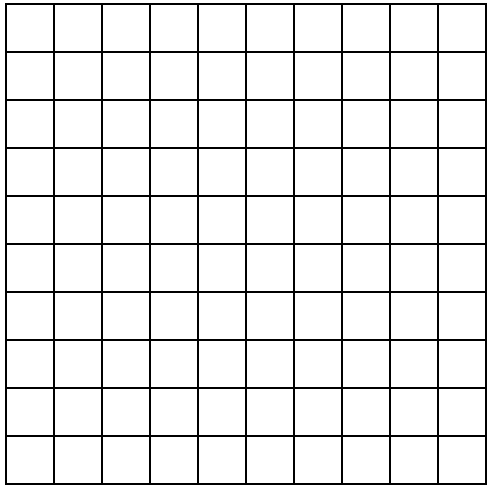
Shots taken: Make or Miss:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Total made/Total taken _____**/20 Decimal** _____ **Percentage**_____

What was our percent?

How do we determine this ratio?



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Bobcats Stats

2005–06 Bobcats Regular Statistics

Player	Free Throws Made/Attempted	Free Throw %	3-Points Made/Attempted	3-Point %
Gerald Wallace	164/224	.614	14/50	.280
Emeka Okafor	82/125	.656	0/0	.000
Brevin Knight	252/314	.803	3/13	.231
Primoz Brezec	164/224	.732	0/2	.000
Raymond Felton	161/222	.725	97/271	.358
Jumaine Jones	88/121	.727	115/335	.343
Kareem Rush	55/77	.714	57/164	.348
Melvin Ely	128/192	.667	0/2	.000
Sean May	49/64	.766	0/5	.000
Matt Carroll	138/168	.821	72/185	.389
Bernard Robinson	106/134	.791	2/21	.095
Alan Anderson	33/41	.805	24/58	.414
Jake Voskuhl	41/60	.683	1/3	.333
Lonny Baxter	12/16	.750	0/0	.000
Kevin Burleson	16/17	.941	10/54	.185

Components of Number Sense Questioning

Quantity

Benchmark fractions (0, 1/2 and 1)
Is 6/14 closer to 0, ½ or 1? What about 17/18? 2/11?
Is 17/33 more or less than ½? How do you know?

Numeration

What does 7/14 mean?
How is 3/10 of a dollar different in our word problem than 3/10 shots?
Which Numeration system tells us what we want to know as the coach?
Why do we sometimes need both?
What does percent mean?

Equality

Is 3/10 of a dollar equal to 6/20 of a dollar?
How can you prove that?
Are 3/5 shots the same thing as 6/10 shots?
How are they the same and how are they different?
In what way are they equal?

Base-ten

Convert from decimal to percent with meaning.
Utilize the decimal system and percentages in order to look at fractions all with the same denominator (1 or 100).
If the decimal form is .52 what is the percent form?
How many powers of ten did you multiply by? Why?

Different forms of a number

What form of the number would the coach want and why?
What information do you get from the fractional form?
What information do you get from decimal form?
Which one do you like better to make comparisons? Why? Or do you usually need both?

Proportional thinking

If I make 12/15 shots, what does it mean that I hit 75%?
What if I took 30 shots, how many would I make at that rate?

Algebraic and Geometric Thinking

Developing arguments and backing them up with logical facts
Creating a flow chart
Proportional reasoning that build into ideas of slope and function.

POWERPOINTS

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Components of Number Sense Overview Unit 4

What's the "Big Idea"?

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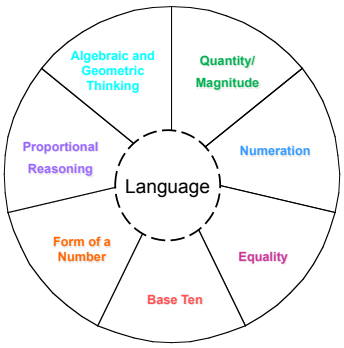

Components of Number Sense

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Language and Math Language

- Is math-language language?
- How can we use some of what we do in reading instruction to improve our mathematics instruction?

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We have to teach math like we teach reading!

- How is math exactly like a mystery novel?
- Do we really try to see the BIG picture?
- How does our chapter in the novel fit the whole story?

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Two stories

- You have a dime (1/10 of a dollar) and seven nickels (7/20 of a dollar).
  - In fractional form show how much money you have all together.
- Two team members record their 3-point shots as 1/10 and 7/20.
  - How did they do as a team?

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Trashketball

The Power to Understanding Percent, Fractions and Decimals

© 2006 Valerie N. Faulkner/Wake County Public School System

Let's Try It!

- Agreements
  - How far will the foul line be from the basket?
  - Are backboards allowable?
  - What else?
- Demo with two players
  - How hot is their shot as a team?
  - Representing their stats

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Trashketball Procedures

- Team members rotate shots.
- After you shoot, you record your result.
- Return to end of line for your next shot
- Team Members stay positive with team members: "Nice shot!" "Nice try." "You're getting better." "We're doing great."
- Stay humble with other teams: "Great percentage!", "Thanks".
- In teams of four, play Trashketball.

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Trashketball Stats

Group Name _____

Group Members: 1._____,(2)_____,(3) _____(4) _____

Shots taken: Make or Miss:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Total made/Total taken _____/20 Decimal _____ Percentage_____

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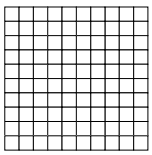
What was our percent?

How do we determine this ratio?



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Trashketball Stats

Group Name _____

Team Members: 1._____,(2)_____,(3)_____,(4)_____

1. Shots taken: Make or Miss:

1 2 3 4 5 6 7 8
Total made/Total taken _____5/8 Decimal _____ Percentage_____

2. Shots taken: Make or Miss:

1 2 3 4 5 6 7 8 9 10 11 12 13 14
Total made/Total taken _____7/14 Decimal _____ Percentage_____

3. Shots taken: Make or Miss:

1 2 3 4 5 6 7 8 9 10 11
Total made/Total taken _____7/11 Decimal _____ Percentage_____

4. Shots taken: Make or Miss:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Total made/Total taken _____7/16 Decimal _____ Percentage_____

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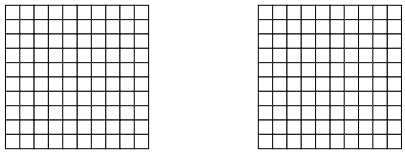
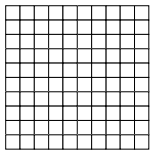


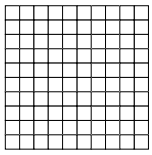
___5/ 8

PICTURE IT!

___7/ 14



___7/ 11



____7/ 16

Bobcats Stats!

A coach can choose any one player to shoot "technical fouls."

If you were the assistant coach of the Bobcats and your team had to shoot technical fouls, who would you suggest to the head coach to shoot the foul shots and why?

Bobcats Stats

2005–06 Bobcats Regular Statistics

Player	Free Throws Made/Attempted	Free Throw %	3-Points Made/Attempted	3-Point %
Gerald Wallace	164/224	.614	14/50	.280
Emeka Okafor	82/125	.656	0/0	.000
Brevin Knight	252/314	.803	3/13	.231
Primoz Brezec	164/224	.732	0/2	.000
Raymond Felton	161/222	.725	97/271	.358
Jumaine Jones	88/121	.727	115/335	.343
Kareem Rush	55/77	.714	57/164	.348
Melvin Ely	128/192	.667	0/2	.000
Sean May	49/64	.766	0/5	.000
Matt Carroll	138/168	.821	72/185	.389
Bernard Robinson	106/134	.791	2/21	.095
Alan Anderson	33/41	.805	24/58	.414
Jake Voskuhl	41/60	.683	1/3	.333
Lonny Baxter	12/16	.750	0/0	.000
Kevin Burleson	16/17	.941	10/54	.185

Trashketball Stats for Story Two

Group Name _____

Group Members: 1._____,(2)_____,(3) _____(4) _____

1. Shots taken: Make or Miss:

1 2 3 4 5 6 7 8 9 10

Total made/Total taken 1/10 Decimal ___.10__ Percentage__10%_

2. Shots taken: Make or Miss:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Total made/Total taken 7/20 Decimal __.35___ Percentage_35%___

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Story Two

- A team has two team members
  - 3-point shots 1/10 and 7/20.
- How did the two of them perform as a team ?

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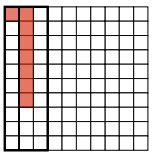
How Did the Team Do Altogether? Story Two

How do we determine this ratio?



__8_ /_30_

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Story One

- You have a dime (1/10 of a dollar) and seven nickels (7/20 of a dollar).
- In fractional form show how much money you have all together.

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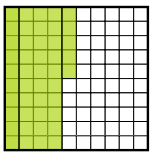
How Do We Represent the Money Problem Story One?

How do we determine this ratio?



_45_ /_100__

1 Dime

7 Nickels

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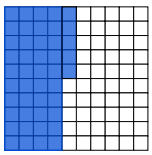
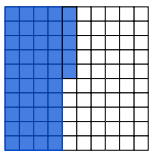
"Move it over two" or Percentage and Decimal relationship using the Components of Number Sense

45% and .45



out of 100



out of 1

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What Do We Ask?

- Quantity/Magnitude
- Numeration
- Equality
- Base Ten
- Form of a Number
- Proportional Reasoning
- Algebraic and Geometric Thinking

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"Arithmetic in American Mathematics Education"

1. What are the steps for solving a mathematical problem in China?
2. What is the American equivalent to Lie Shi?
3. Using 10 sticks model a figure and then talk about the mathematical expressions that exist within the visual.
4. What are the key points that Ma makes in this article?

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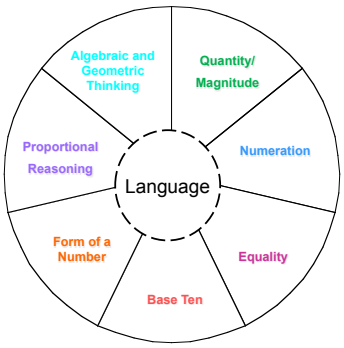

Components of Number Sense

Public Schools of North Carolina

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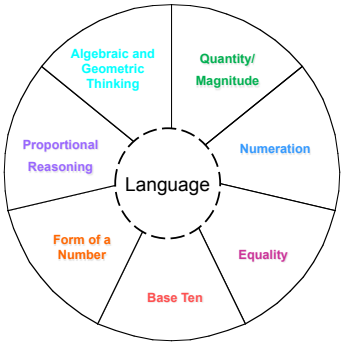

Components of Number Sense

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Assignments from Units 3 and 4

- Due Day 3
- Required Reading:
  - "Subitizing" Doug Clements
  - Continue to read Knowing and Teaching Elementary Mathematics Liping Ma

Unit 5

REQUIRED READING

HANDOUTS

The Number Knowledge Test: Administration and Scoring

The Number Knowledge test is an oral test. It is administered, individually, to each child and it requires oral responses from each child. Children are not permitted to use paper and pencils to figure out the answers. Although this is not an issue at the kindergarten level, older children often request paper and pencils guessing, correctly, that the problems would be easier to solve if children did not have to figure them out "inside their heads." To measure intuitive knowledge, however, it is important that children DO figure things out in their heads. At the kindergarten level, the test takes about 5 to 10 minutes to complete.

Before actually administering the test, you will need to prepare your own test kit. This entails creating the visual props required for some items, ensuring you have sufficient scoring sheets for each child you will be testing, and assembling all materials so you can manipulate them easily and comfortably. It is strongly recommended that you practice giving the test to a friend or colleague before using it with children in your class. This will ensure that test administration and scoring is automatic and it will allow you to focus all your attention on the child's responses, problem-solving strategies, and other behaviors (e.g. expressions of frustration) that provide indications of his or her level of understanding.

Suggestions for administering the test -

• Plan a time when the other children in your class are occupied, so you will not be interrupted.

• Choose your location carefully so that materials in the surroundings, such as a number line, will not provide visual aids.

• Prepare the child for the test by explaining that the questions will be easy at first but they will get harder and harder. You don't expect them to know all the answers. Some of them are even challenging for older children to get right!

• For children who are 6 or younger, start at the preliminary level and continue testing until the child has not passed sufficient items at any level to progress to the next level, as specified on the test form. Whenever possible, enter the child's answer on the scoring form and mark each item as passed or failed in such a way that failures are not apparent to the child.

• For children who are 8 and older, you can omit the preliminary item and Level 0 and start testing at Level 1. As a general rule, always start testing at a level that is at least two years below the age of the child you are testing. This will ensure that children experience success on their beginning level of the test and provide an index of their baseline knowledge.

• Administer all items at each level that you progress to (unless the child is terribly frustrated) in order to determine the child's maximum potential.

• Provide frequent reinforcement throughout, for example by saying "Good" after each response) without actually telling children whether their answers are right or wrong.

• If a child's strategy is not apparent, you may wish to ask, "How did you figure that out?" and record the strategy used or reported on the scoring sheet. Directions for coding strategy use on the scoring sheet are provided below.

Coding Strategy Use -

For selected items on the test (such as items 1, 3, and 7 at Level 1; items 1, 2, and 8 at Level 2; items 1, 5, and 6 at Level 3), additional information about a child's number knowledge can be gained by coding strategy use. If the strategy used is not apparent in children's responses, ask them how they figured out the answers (i.e. "How did you figure that out?"). Based on the child's explanation, mark one of the following on the test record form:

CU = Count up from "one". Circle CU if the child uses a counting up from "one" strategy for solving addition problems.

CO = Count on. Circle CO if the child enters the counting sequence at the point of one of the addends and then counts on, one number at a time.

R = Retrieval. Circle R if the child says that she figured out the answer in her head (e.g. "I just knew it", "My brain told me", "I learned that").

Scoring the test

One point is assigned for each item passed at Levels 0, 1, 2 and 3. For all two-part items, both (a) and (b) must be passed to earn one point. If testing commences at Level 1 (for children who are at least 8 years old), points are automatically assigned for Level 0 items.

A total raw score for each child on this test can be computed by summing the number of points the child received across all levels of the test. A chart to convert raw scores to developmental levels scores is provided below and can be used if you find this information useful. It is important to note in this context that a developmental level describes the knowledge or the performance of an abstract entity: the "average" child in our culture. In practical terms, this means that about 60% of the U.S. children who have taken this test perform at the level indicated. About 20% perform somewhat better, some as high as the next whole level up. About 20% perform somewhat worse, some as low as the next whole level down. For this reason, developmental level scores provide only a rough index of a child's knowledge or growth.

Developmental Level Conversion Chart

Raw Score	Developmental Level Score	C.A. Equivalents
1-3	-0.5	2-3 years
4-6	0.0	3-4 years
7-8	0.5	4-5 years
9-14	1.0	5-6 years
15-19	1.5	6-7 years
20-25	2.0	7-8 years
26-28	2.5	8-9 years
29-30	3.0	9-10 years

Interpreting the test -

Although test results can be used to compare the performance of a particular child to the rest of the class or to compare your class to national averages, a far more important use of the test is for instructional planning. When used at the beginning of the year, the test can give you valuable information on the sorts of number knowledge each child brings to the learning situation, and the strategies each child has available to make sense of number problems. You can use this information to decide which Number Worlds activities each child might benefit from and you can use it to form small groups for a Number Worlds exploration. You may wish to group children so that all children in the group are at a similar level in their number understanding or you may wish to group children in such a fashion that more knowledgeable children can function as group leaders or "mini-teachers."

When used at the end of the year, the test can give you a measure of each child's learning progress, knowledge growth, and readiness for mathematics learning at the next grade level. The test can also be used in the middle of the year if an interim measure of children's knowledge is desired; for example, you might want to configure new learning groups for a continuing exploration of Number Worlds.

Test results can also be used to help you determine the strengths and weaknesses of your class as a whole; for example, you can quantity the percentage of children who pass or fail particular items of the test. This information can be invaluable for instructional planning. If you discover that most children in your class have an easier time telling you which of two numbers is bigger than which of two numbers is smaller, you may wish to devote extra time to the count-down "Blast Off" activities that are included in each Number Worlds land.

Finally, as with individual children, the information you gather at the beginning of the year for your class as a whole can be used at the end of the year to give you a picture of the progress your class has made as a group. When you compare beginning- to end-of-the-year results, you may obtain a well-deserved sense of accomplishment as you identify areas of growth. If growth is not as great as you expected or hoped for in some areas, you can use these results to modify or adapt your instructional planning for the next school year.

Number Knowledge Test Record

Total Score

Name	D.O.B
Date	
School	
Examiner	

Preliminary 1-10

Level 0 (3 or more correct, go to next level)

		Notes
1. Count (3)		
2a. 5 vs. 2	2b. 3 vs. 7	
3a. 2 vs. 6	3b. 8 vs. 3	
4. Count Y (4)		
5. Count R (8)		

Level 1 (5 or more correct, go to next level)

1. 4 + 3=	CU CO R	
2. 7 + 1 =	CU CO R	
3. 7 + 2 =	CU CO R	
4a. 5 or 4	4b. 7 or 9	
5a. 8 or 6	5b. 5 or 7	
6a. 5 : 6 or 2	6b. 7: 4 or 9	
7. 2 + 4 =	CU CO R	
8. 8 - 6 =		
9a. 8 5 2 6	9b. 8 5 2 6	

Level 2 (5 or more correct, go to next level)

1. 49 + 5 =		
2. 60 - 4 =		
3a. 69 or 71	3b. 32 or 28	
4a. 27 or 32	4b. 51 or 39	
5a. 21 25 or 18	5b. 28: 31 or 24	
6. #'s between 2 and 6		
7. #'s between 7 and 9		
8. 12 + 54 =	Add CO	
9. 47 - 21 =		

Preliminary

Let's see if you can count from 1 to 10. Go ahead.

Number Knowledge Test Level 0 (4-year-old level)

1. (Place 3 Counters in a row in front of the child.)
Can you count these Counters and tell me how many there are?

2a. (Show stacks of counters, 5 vs. 2, same color)
Which pile has more?
2b. (Show stacks of counters, 3 vs. 7, same color)
Which pile has more?

3a. This time, I'm going to ask you which pile has less.
(Show stacks of counters, 2 vs. 6, same color.)
Which pile has less?
3b. (Show stacks of Counters, 8 vs. 3, same color.)
Which pile has less?

4. I'm going to show you some Counters.
(Show a line of 3 blue and 4 white Counters in a row, as follows: BWBWBWW.)
Count just the white Counters and tell me how many there are.

5. (Pick up all the Counters from the previous question. Show a mixed array—not a row—of 7 white and 8 blue Counters) Here are some more Counters. Count just the blue counters and tell me how many there are.
*Go to Level 1 if 3 or more correct
In order to receive credit an item that has an a and b, both a and b must be correct to be counted in the score.

Number Knowledge Test Level 1 (6-year-old level)

1.If you had 4 chocolates and someone gave you 3 more, how many chocolates would you have altogether?

2.What number comes right after 7?

3.What number comes two numbers after 7?

4a Which is bigger: 5 or 4 ?
4bWhich is bigger: 7 or 9 ?

5a This time, I'm going to ask you about smaller numbers
Which is smaller: 8 or 6?
5b Which is smaller: 5 or 7?

6a Which number is closer to 5: 6 or 2?
(show visual array after asking)
6b Which number is closer to 7: 4 or 9?
(show visual array after asking)

7. How much is 2 + 4 ? (okay to use fingers)

8. How much is 8 take away 6 ? (okay to use fingers)

9a When you are counting, which of these numbers do you say first? (show visual array 8526-ask child to point and say each numeral)
9b When you are counting, which of these numbers do you say last?

*Go to level 2 if 5 or more correct

In order to receive credit an item that has an a and b, both a and b must be correct to be counted in the score.

Number Knowledge Test Level 2 (8-year-old level)

1. What number comes 5 numbers after 49?

2. What number comes 4 numbers before 60?

3a. Which is bigger: 69 or 71
3b. Which is bigger: 32 or 28?

4a. This time I'm going to ask you about smaller numbers.
Which is smaller: 27 or 32?
4b Which is smaller: 51 or 39?

5a Which number is closer to 21: 25 or 18?
(Show visual array after asking the question)
5b Which number is closer to 28: 31 or 24?
(Show visual array after asking the question)

6. How many numbers are there in between 2 and 6?
(Accept either 3 or 4)

7. How many numbers are there in between 7 and 9?
(Accept 1 or 2)

8. (Show visual array 12 54.) How much is 12+54?

9. (Show visual array 47 21.) (Use term take away)
How much is 47-21?

*Go to Level 3 if 5 or more correct
In order to receive credit an item that has an a and b, both a and b must be correct to be counted in the score.

Number Knowledge Test Level 3 (10-year-old level

1. What number comes 10 numbers after 99?

2. What number comes 9 numbers after 99?

3a. Which difference is bigger, the difference between 9 and 6 or the difference between 8 and 3?

3b. Which difference is bigger, the difference between 6 and 2 or the difference between 8 and 5?

4a. Which difference is smaller, the difference between 99 and 92 or the difference between 25 and 11?

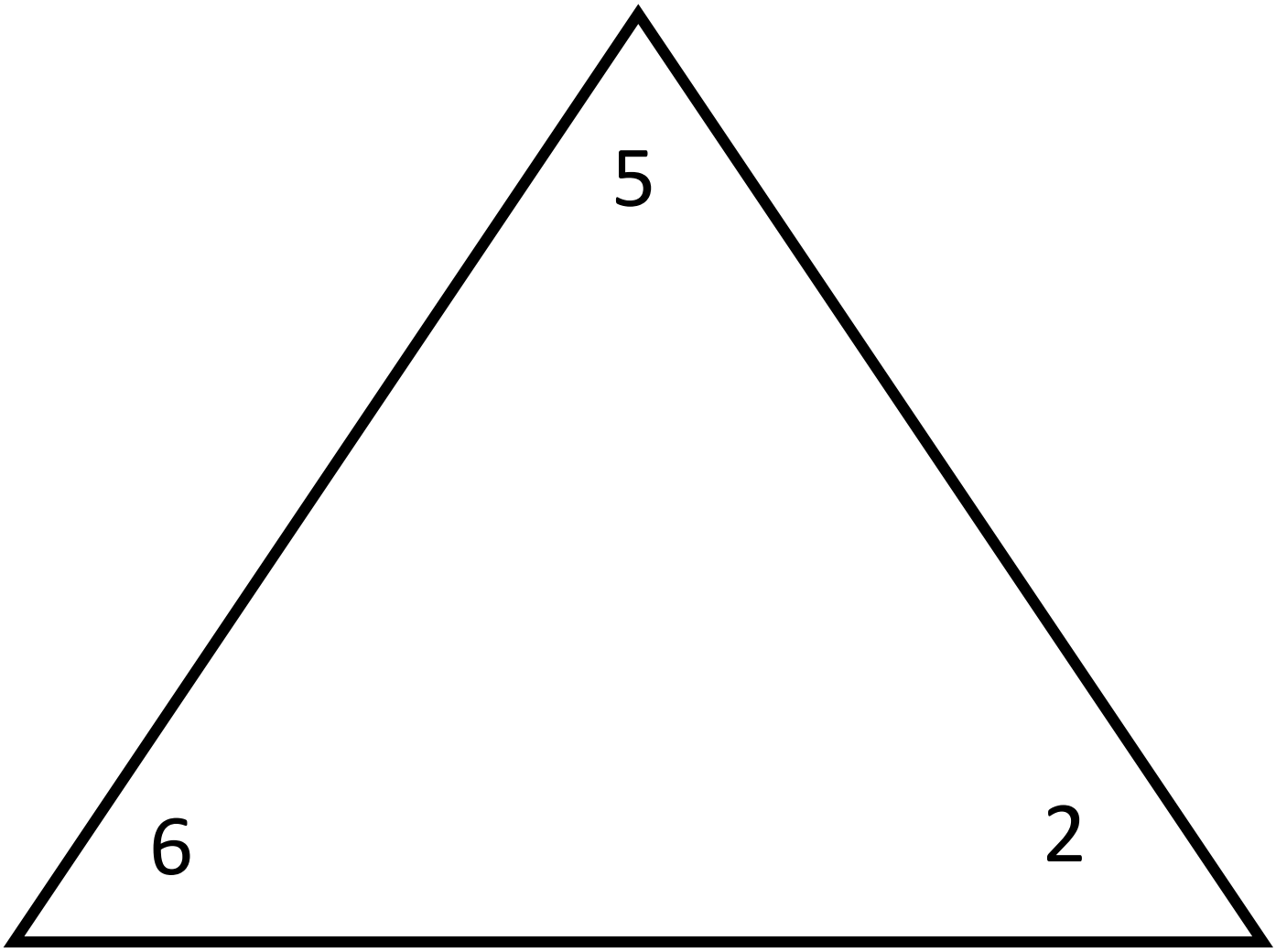
4b. Which difference is smaller, the difference between 48 and 36 or the difference between 84 and 73?

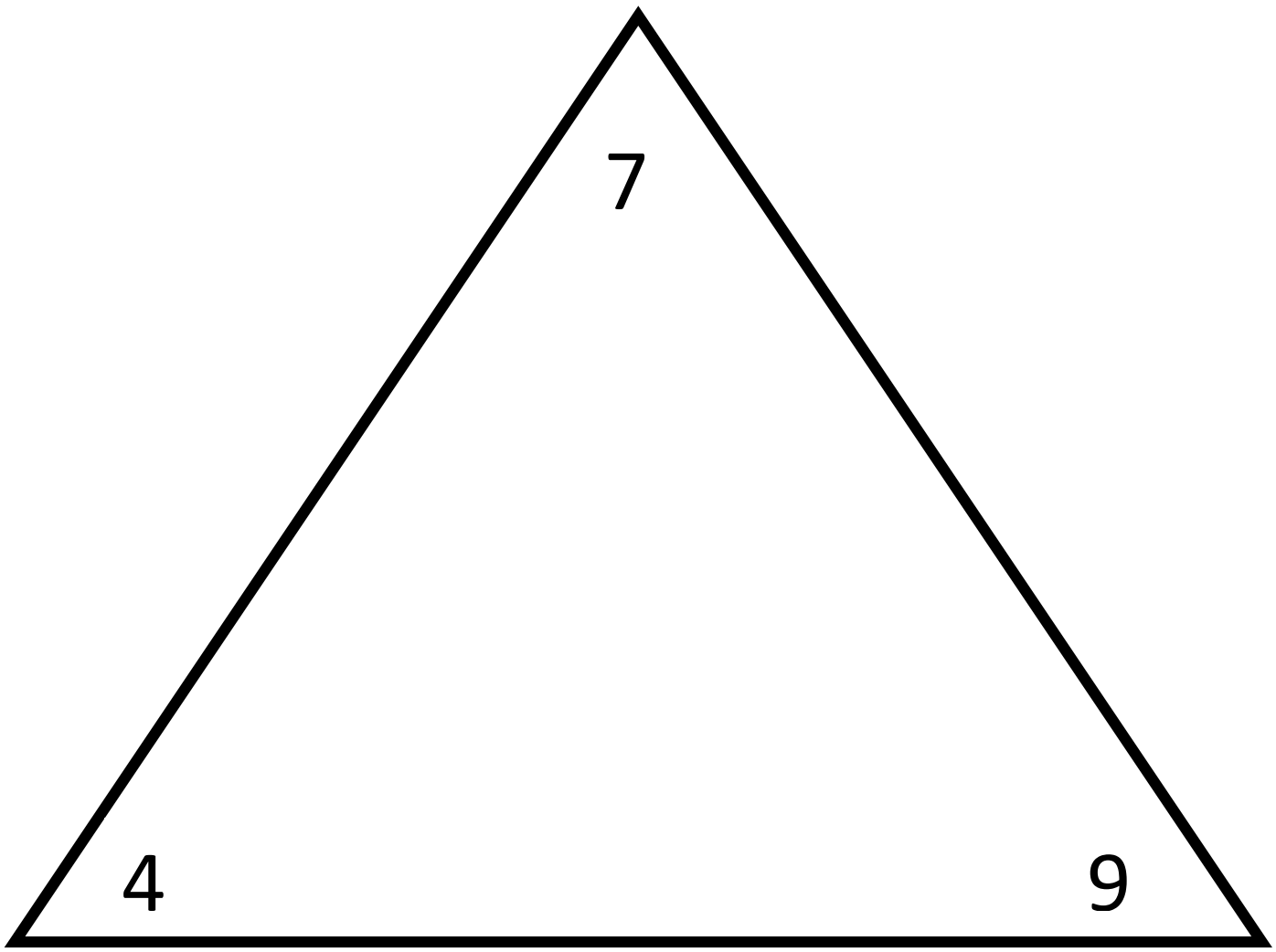
5. (Show visual array of 13 and 39.)
How much is 13 + 39?

6. (Show visual array of 36 and 18.)
How much is 36 – 18?

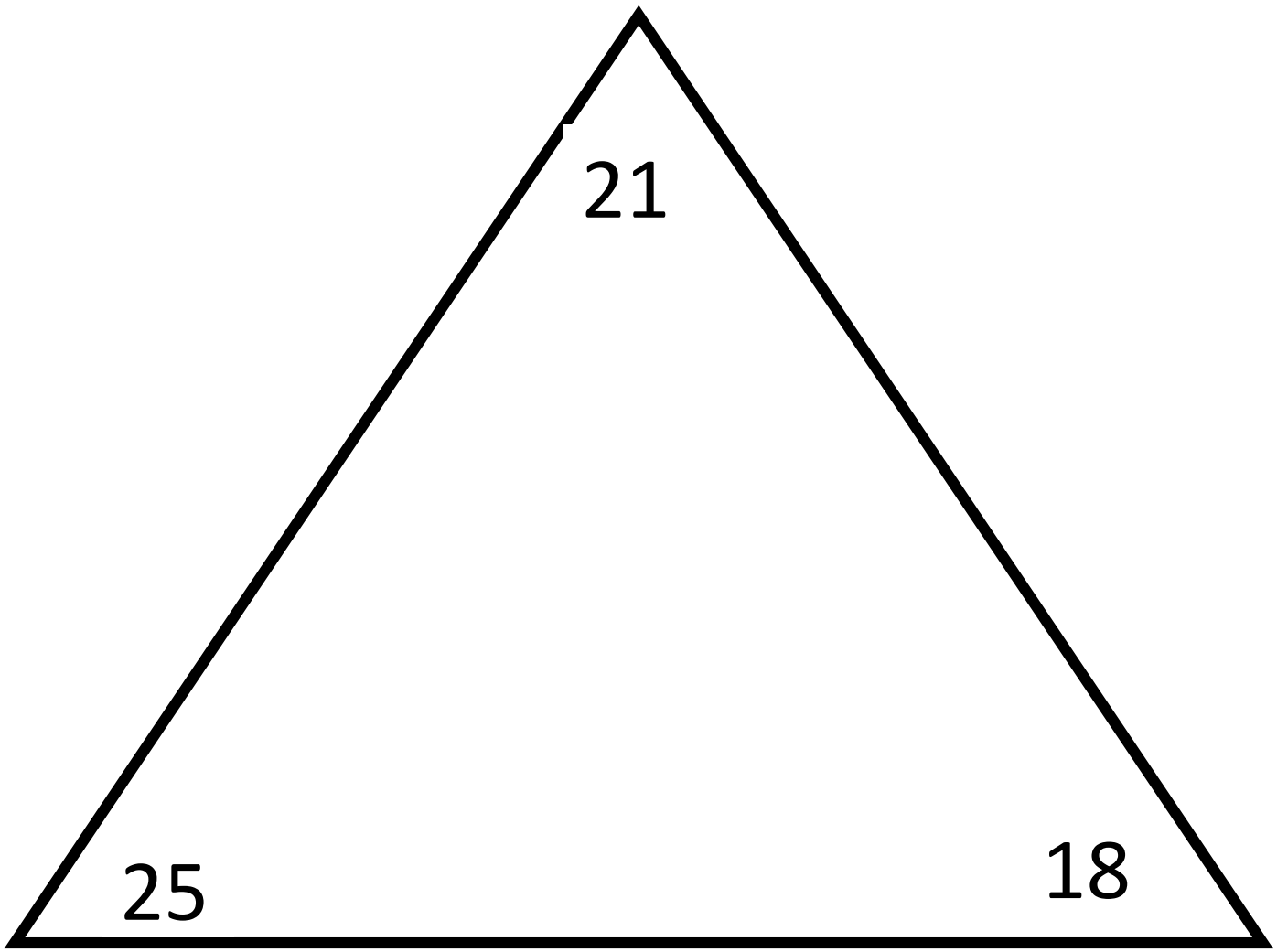
7. How much is 301 take away 7?

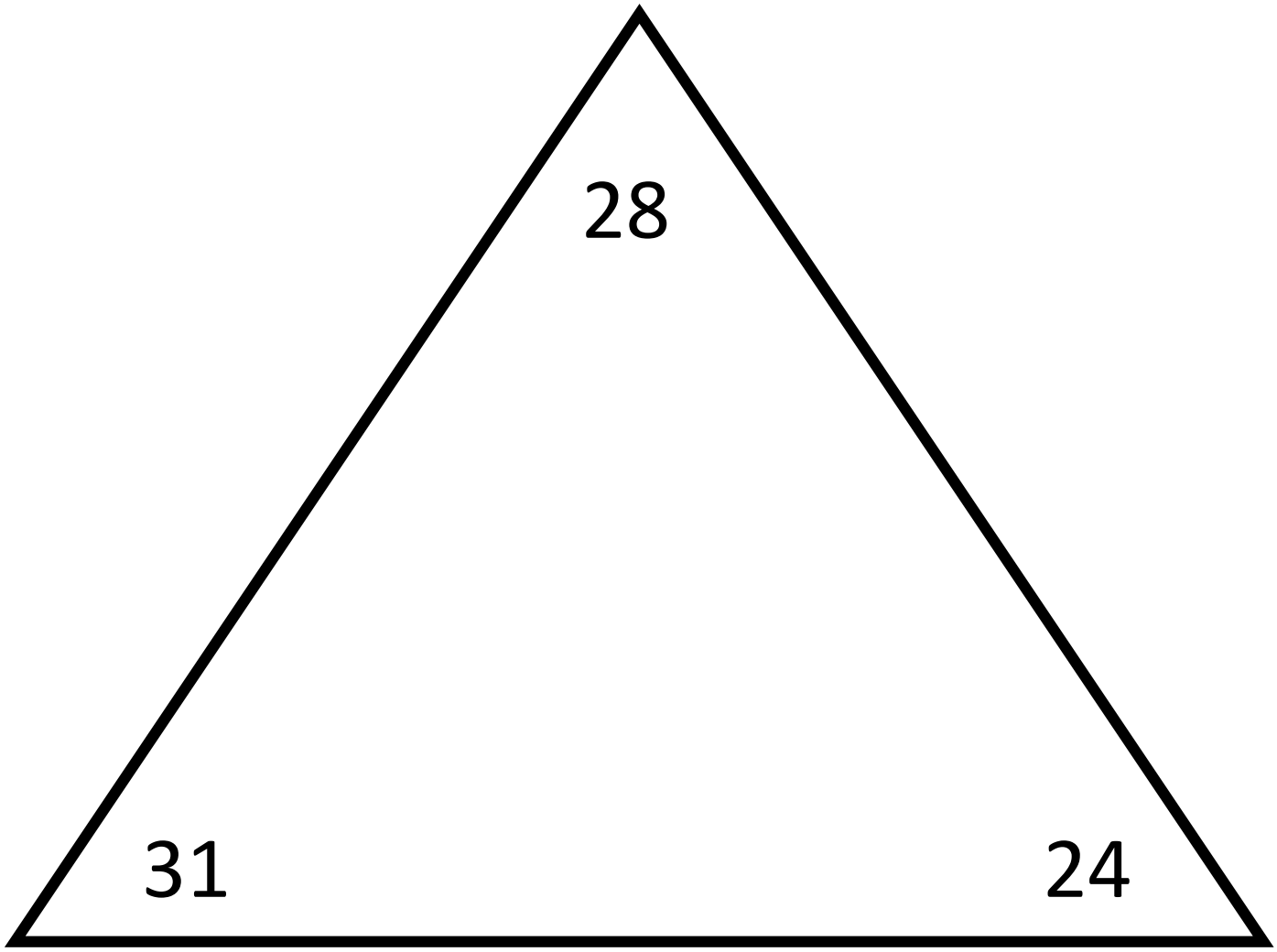
In order to receive credit an item that has an a and b, both a and b must be correct to be counted in the score.


5
6
2


7
4
9

8 5 2 6


21
25
18


28
31
24

12	54

47	21

13	39

36	18

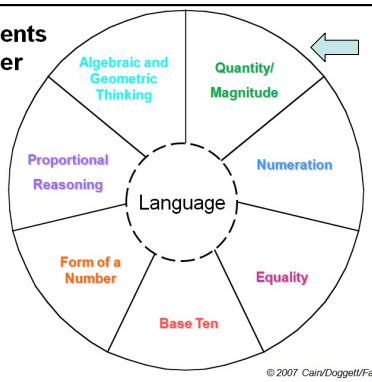
POWERPOINTS

Unit 5
Quantity / Magnitude

Defining the Concept

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Components of Number Sense



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Gellman and Gallistel's (1978) Counting Principles

- 1-1 Correspondence
- Stable Order
- Cardinality
- Abstraction
- Order-Irrelevance

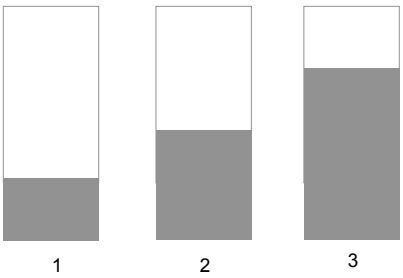
Geary and Hoard, Learning Disabilities in Basic Mathematics from Mathematical Cognition, Royer, Ed.

Abstraction

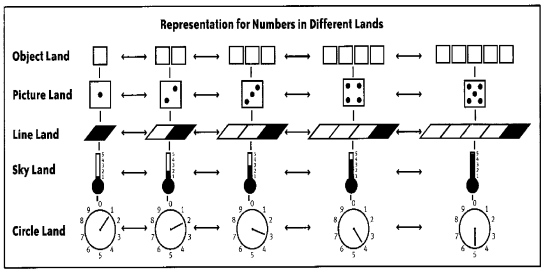
- 3 ones and 2 ones
- 3 tens and 2 tens
- 3 tens and 2 ones
- 3/6 and 2/6
- 3/6 and 2/5

- 3X and 2X
- 3Y and 2Y
- 3X and 2Y

Use of the Number Line



Different Forms of a Number-- Linking to Magnitude to Number Lines



Number Worlds Griffin

Diagnosis

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Early Indicators of Problems with Quantity/Magnitude:

- Early sequencing skills are delayed
- Visual spatial issues
- Child has difficulty rote counting
- Child has difficulty with rational counting (1 to 1 correspondence)

Adapted from Harding & Harding

Number Knowledge Test

- http://clarku.edu/numberworlds
- Video of the Number Knowledge Test Administration

Preliminary

Let's see if you can count from 1 to 10. Go ahead.

Number Knowledge Test Level 0 (4-year-old level)

____ 1. Can you count these Counters and tell me how many there are?
(Place 3 Counters in a row in front of the child.)

____ 2a. (Show stacks of counters, 5 vs. 2, same color)
Which pile has more?

____ 2b. (Show stacks of counters, 3 vs. 7, same color)
Which pile has more?

____ 3a. This time, I'm going to ask you which pile has less.
(Show stacks of counters, 2 vs. 6, same color.)
Which pile has less?

____ 3b. (Show stacks of Counters, 8 vs. 3, same color.)
Which pile has less?

____ 4. I'm going to show you some Counters.
(Show a line of 3 blue and 4 white Counters in a row, as follows: BWBWBWW.)
Count just the white Counters and tell me how many there are.

____ 5. (Pick up all the Counters from the previous question. Show a mixed array—not a row—of 7 white and 8 blue Counters)
Here are some more Counters.
Count just the blue counters and tell me how many there are.

*Go to Level 1 if 3 or more correct.

In order to receive credit an item that has an a and b, both a and b must be correct to be counted in the score.

Number Knowledge Test Level 1(6-year-old level)

__ 1. If you had 4 chocolates and someone gave you 3 more, how many chocolates would you have altogether?
__ 2.What number comes right after 7?
__ 3.What number comes two numbers after 7?
__ 4a Which is bigger: 5 or 4 ?
4bWhich is bigger: 7 or 9 ?
__ 5a This time, I'm going to ask you about smaller numbers
Which is smaller: 8 or 6?
5b Which is smaller: 5 or 7?
__ 6a Which number is closer to 5: 6 or 2?
(show visual array after asking)
___6b Which number is closer to 7: 4 or 9?
(show visual array after asking)
__ 7. How much is 2 + 4 ? (okay to use fingers)
__ 8. How much is 8 take away 6 ? (okay to use fingers)
__ 9a When you are counting, which of these numbers do you say first? (show visual array 8526-ask child to point and say each numeral)
___9b When you are counting, which of these numbers do you say last?

*Go to level 2 if 5 or more correct,

In order to receive credit an item that has an a and b, both a and b must be correct to be counted in the score.

.

Number Knowledge Test Level 2 (8-year-old level)

_____ 1.What number comes 5 numbers after 49?
_____ 2.What number comes 4 numbers before 60?
_____ 3a.Which is bigger: 69 or 71 ?
3b.Which is bigger: 32 or 28 ?
_____ 4a.This time I'm going to ask you about smaller numbers.
Which is smaller: 27 or 32 ?
4b Which is smaller: 51 or 39 ?
____ 5a Which number is closer to 21: 25 or 18?
(Show visual array after asking the question)
5b Which number is closer to 28: 31 or 24?
(Show visual array after asking the question)
____ 6. How many numbers are there in between 2 and 6?
(Accept either 3 or 4)
____ 7. How many numbers are there in between 7 and 9?
(Accept 1 or 2)
____8. (Show visual array 12 54.) How much is 12+54?
____9. (Show visual array 47 21.) How much is 47-21?
(use term take away)

*Go to Level 3 if 5 or more correct.

In order to have an Item that has a and b, both a and be must be correct to be counted in the score.

Number Knowledge Test Level 3 (10-year-old level

____ 1. What number comes 10 numbers after 99?
____ 2. What number comes 9 numbers after 99?
____ 3a. Which difference is bigger, the difference between 9 and 6 or the difference between 8 and 3?
____ 3b. Which difference is bigger, the difference between 6 and 2 or the difference between 8 and 5?
____ 4a. Which difference is smaller, the difference between 99 and 92 or the difference between 25 and 11?
____ 4b. Which difference is smaller, the difference between 48 and 36 or the difference between 84 and 73?
____ 5. How much is 13 + 39? (Show visual array of 13 and 39.)
____ 6. How much is 36 – 18? (Show visual array of 36 and 18.)
____ 7. How much is 301 take away 7?

In order to receive credit an item that has an a and b, both a and b must be correct to be counted in the score.

Developmental Level Comparison Chart

Raw Score	Developmental Level Score	C.A. Equivalents
1-3	-0.5	2-3 years
4-6	0.0	3-4 years
7-8	0.5	4-5 years
9-14	1.0	5-6 years
15-19	1.5	6-7 years
20-25	2.0	7-8 years
26-28	2.5	8-9 years
29-30	3.0	9-10 years

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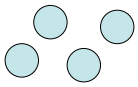
Where Research Meets the Road

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Subitizing



This is a critical skill and may lay underneath early math number sense difficulties with addition and subtraction.

Doug Clements, Julie Sarama

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Number Sense and Instructional Choices

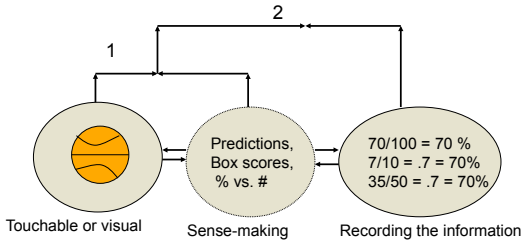
Number Sense and Instructional Choices

"Subitizing"

Doug Clements

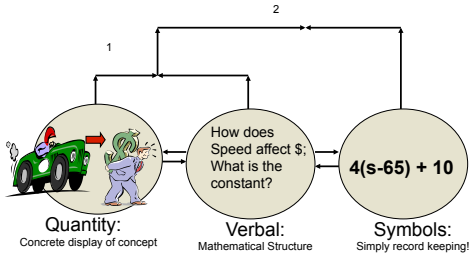
- What is subitizing?
- What is the difference between perceptual and conceptual subitizing?
- What factors influence the difficulty level for students in subitizing?
- What are the implications for teaching?
- What are some strategies that teachers can use to promote subitizing?

Example: Percentages



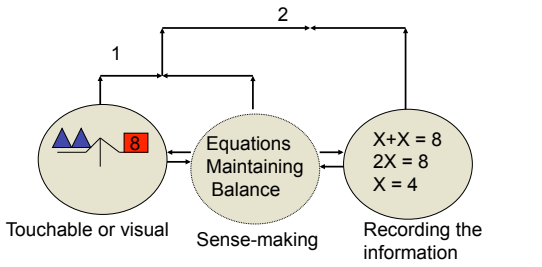
V. Faulkner and NCDPI Task Force adapted from Griffin

Even Algebra can fit this mold!



V. Faulkner and NCDPI Task Force adapted from Griffin

Example: Algebra—Solving Equations



V. Faulkner and NCDPI Task Force adapted from Griffin and Borenson: Hands on Equations

Research

Mercer's research supports the idea of teaching students with disabilities with

C-R-A approach

Concrete

Representational

Abstract

Hands-on Equations provides excellent Concrete/Representational support for students.

Borenson Video

Level #1: Lessons #1-#7

Solve equations such as:

2X + X + X + X + 2 = 2X + 10

and

2(X+4) + X = X + 16

Materials for level #1

- Eight Blue Pawns
- Two Red Cubes, Numbered 0-5
- Two Red Cubes, Numbered 5-10
- A paper drawing of Balance Scale

Equality

A "Human Scale" can be used to demonstrate the idea behind EQUALITY

Equality is a Big Idea in algebra and needs to be reinforced again and again

Equality

$3 = 3$

$3+4 = 3+4$

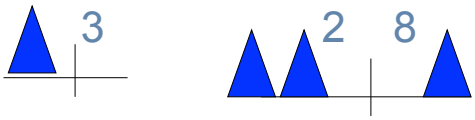
$7 = 7$

$5 = 5$

$5\cdot(1/5) = 5\cdot(1/5)$

$1 = 1$

Solve by Trial and Error
Beginning with Modeling and Think Aloud



Lesson #1

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Blue Pawn is named "X"

X + 4 = 2X + 3

- ☞ Physically Represent Equation
- ☞ Solve by Trial and Error: for us that will generally mean **Model** how you would solve by trial and error. **Think Aloud.**

Lesson #2

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Legal Move and Equality

- ☞ Introduce the Concept of a "**Legal Move**"
- ☞ Review the idea of **Equality**, show that it is a "legal move" for a mathematical reason

4X + 2 = 3X + 9

Lesson #3

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$$X + 2 + 2X = X + 10$$

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Use Legal Move with Numbered Cubes

$$2X + 4 = 10$$

Lesson #4

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Use Legal Move with Numbered Cubes and Pawns

$$X + 2X + 14 = 5X + 2$$

$$14 = 2X + 2$$

$$12 = 2x$$

Lesson #4

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$$4X + 5 = 2X + 13$$

Lesson #4

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Express Subtraction by the Removal of Pawns

$$5X - 3X + 2 = X + 5$$

$$2x + 2 = X + 5$$

$$X = 3$$

3+3+3+3+3-3-3-3+2=3+5

8=8 (It checks!) ✓

Lesson #5

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$$3X + 4 = 5X-3X + 9$$

Lesson #5

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Distributive Property

2 (X+3) = X + 8

Lesson #6

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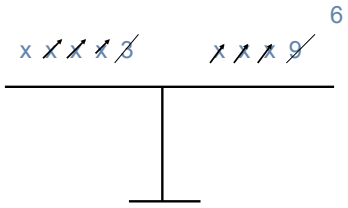
Pictorial Notation

4x + 3 = 3x + 9



X = 6 Check 27 = 27 ✓

Lesson # 7

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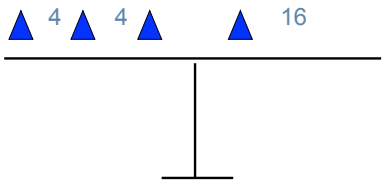
Solve Using Representation

2 (x + 4) + x = x + 16



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Quantity/Magnitude

- Size
- Weight
- Number in a set
- Length
- Capacity of a container
- Value of an object
- Area
- Time
- Likelihood of an occurrence

Classroom Applications

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Quantity/Magnitude in action Utilizing the Number Line to develop understanding of fractions

Beyond Slices of Pizza

Mathematical Problem

At Food Lion, butter costs 65 cents per stick.
This amount is 2 cents less per stick, than butter at Lowes.
If you need to buy 4 sticks of butter, how much will you pay at Lowes?

Adapted from Hegarty, Mayer, Green (1992)

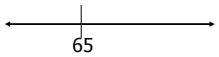
Translating

Converting a sentence into a mental representation

- Assignment Sentence! Easy Peasy!

At Food Lion, butter costs 65 cents per stick.

So --- Food Lion = 0.65



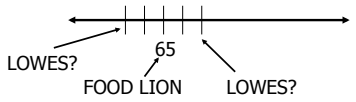
Relational Sentences: Uh – oh…

This amount is 2 cents less per stick, than butter at Lowes.

Say What?

Building a Mental Model of the Problem Situation

The hard part of this problem is NOT the multiplication. It is figuring out the cost of butter at the two stores!



Younger than, fewer than, fewer, less, less than, more than? Help!

"More than means add"

"Less than means subtract"

"For less than take the two numbers and subtract but switch the two numbers"

Cultural Problem

The problem is that almost all of the strategies we learned in math pretend that life is filled with nothing but assignment sentences!

The only thing that can really help is to Deeply Develop and Understand Arithmetic Relationships – VISUALLY AND STRUCTURALLY.

This allows us to make sense of the relational sentences.

Develop Vocabulary in CONTEXT

The number line IS the context

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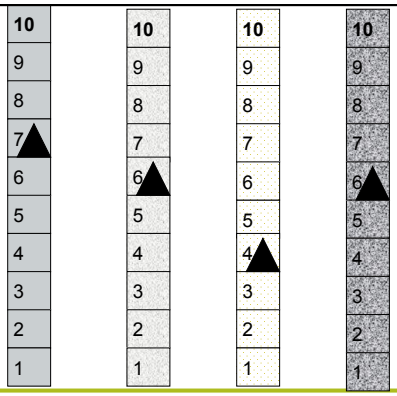

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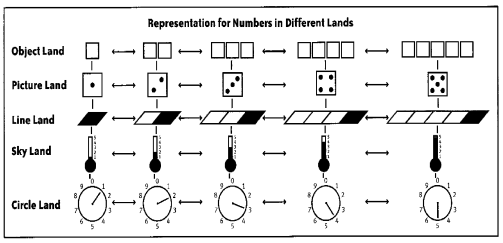
Number Worlds



Vocabulary Development with Meaning - Not "disembodied"!

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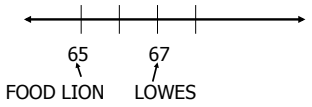
Integrating

Building a mental model of the problem situation

Butter problem seen as "total" cost

(Food Lion Butter + 2) x 4 = Cost at Lowes



Planning

- Devising a plan for how to solve the problem.
  - First, add 2 cents to 65 cents and then multiply the result by 4.

Planning

Structural similarity vs. Surface similarity

Structurally similar

- Money problem (100) and basketball percentage (100)
- Speed of a car (slope) and salary problem (slope)

Surface similarity

- All basketball problems together
- All transportation problems together

Executing

- Carrying out the plan

65 + 2 = 67
67 x 4 = 268
268 cents = $2.68

Executing

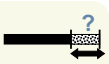
Those who learned with a multi-representational approach were able to generalize the skill better.

Structures of Subtraction

The Classic "Take away" (How many are left?)
You've got some amount and "take away" from that amount. How many are left?

Comparison (Difference between? Who has more?)
You compare to see who has more or less?

Deficit/Missing amount (What's missing?)
You need some more to get where you want to be. What is the missing amount?

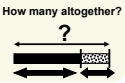


Structures of Addition

Join and Part-Part Whole

– There is something, and you get more of it?

– There are two kinds, how many all together?

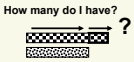


Start Unknown

Some are given away, some are left, how many were there to start?

Compare--total unknown

I know one amount and I have some amount more than that. How many do I have?



Adapted from Carpenter, Fennema, Franke, Levi and Empson, 1999 p. 12 in Adding it Up, NRC 2001.

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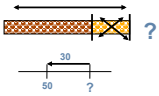
Addition
Start Unknown

Julie had a bunch of fruit. She gave away 30 oranges and she still has 50 pieces of fruit left. How many pieces of fruit did she have to start with?



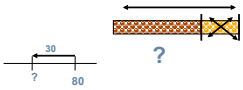
left + gave away =start

gave away + left = start

addend + addend = Sum(Total)

Subtraction
Classic "take-away"

Julie had 80 pieces of fruit. She gave away 30 oranges. How many pieces of fruit did she have left?



start - gave away = left

start – left = gave away

Sum(Total) – addend = addend
minuend – subtrahend = difference

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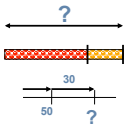
Addition
Join or Part/Part -Whole

Julie had 50 apples and then bought 30 oranges. How many pieces of fruit does she have now?

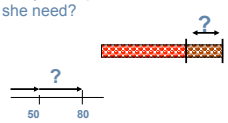


part + other part = whole

addend + addend = sum

Subtraction
Deficit/ Missing Amount

Julie wanted to collect 80 pieces of fruit for a food drive. She already has 50 apples. How many more pieces of fruit does she need?



whole – part = other part

whole – part accounted for = part needed

whole – part = difference

minuend – subtrahend = difference

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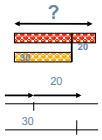
Addition
Compare: Total unknown

Julie had 30 oranges and some apples. She had 20 more apples than oranges. How many apples does she have?

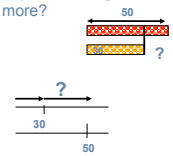


Amount of one set + the difference between two sets = amount of second set

Addend + addend = sum total (of unknown set)

Subtraction
Compare: difference unknown

Julie had 50 apples and 30 oranges. Does she have more apples or oranges? How many more?



Amount in one set – amount of an other set = difference between sets

Sum total (needed) – amount of one set (have) = difference

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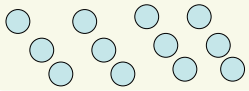
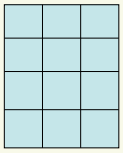
STRUCTURE:
3 Types of Multiplication: 4 x 3

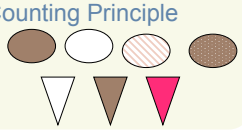
Repeated Addition



Array/ row-column



Counting Principle



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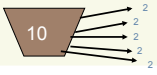
Division Structures

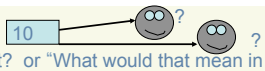
Measurement/Repeated Subtraction

"How many 2s can I get out of 10?"



If I have 10 cups of beans and I give out 2 cup portions, how many servings will that provide?

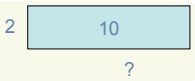
Partitive/Unitizing/Fair Shares



"How many would one person get? or "What would that mean in relation to 1?"

If 2 people find $10 how much will each person get ?

Product/Factor



"If I have an area of 10 and one side is 2, how long is the other side?"

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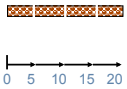
Multiplication
Repeated Addition

Julie had 4 baskets with 5 pieces of fruit in each basket. How many pieces of fruit does she have?

5 + 5 + 5 + 5



How many is 4 5s?

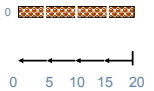
# of Groups * Objects in group = Total objects

Factor * Factor = Product

Division
Repeated Subtraction/ Measurement

Julie has 20 pieces of fruit. She wants to eat 5 pieces of fruit a day. How many days can she eat her fruit?

-5 - 5 - 5 - 5 ← 20



How many 5s can you get out of twenty?

Total ÷ portions = servings

Product ÷ factor = factor

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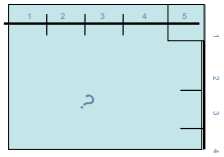
Multiplication
Array/Row-Column (Area/Side Length)

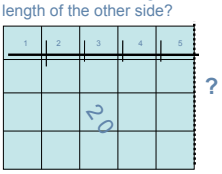
Julie has a rectangular surface she wants to cover with square unit tiles. The length of one side is 5 units long and the length of the other side is 4 units long. How many pieces of tile does Julie need?



Linear side · Linear side = Area of Rectangle
Row · Column = Total
Factor · Factor = Product of Area

Division
Product/Factor (Area/Side Length)

Julie has a rectangular surface that is 20 square units. The length of one side is 5 units long. What is the length of the other side?



Area of Rectangle ÷ Linear side = Other linear side
Total ÷ Column = Row
Total ÷ Row = Column
Product ÷ Factor = Factor

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Multiplication
Fundamental Counting Principle

Julie packed 4 pair of jeans and 5 shirts for her trip. How many different unique outfits can she make?

Total outfits?

This is also an excellent model for probability:
Julie has four dice in different colors: blue, red, green and white. If she picks one die at random and then rolls it, what are the chances that she would have rolled a blue and a 5?

P(Blue,5)?

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Division
Partitive/Unitizing/Fair Shares

Julie is packing her suitcase for a trip. She is planning her outfits for the trip and will wear one shirt and one pair of jeans each day. She brought 5 shirts. How many pairs of jeans must she bring if she needs 20 unique outfits?

This model is the way students first learn division, through 'fair shares'? How many will each one person get?

It is also the structure for a Unit Rate: 20 per every 4, how many per 1?

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Assignment Sentences

- Food Lion = 65 cents
- Let x = 5
- Laura's Age = 21
- The 727 travels at 250 mph
- 1 pound = 16 ounces

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Relationship Sentences

- The price of Food Lion's butter is 2 cents less than the price of Lowe's butter per stick.
- y = x - 1
- Laura is 2 years younger than Valerie.
- The 757 travels 56 mph faster than the 727.
- The package weighs 5 ounces less than the pound of hamburger.

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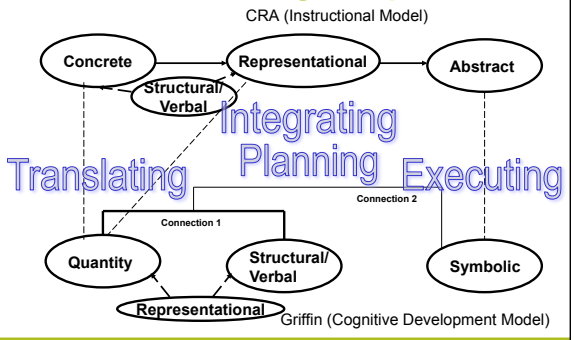
Translation Exercise

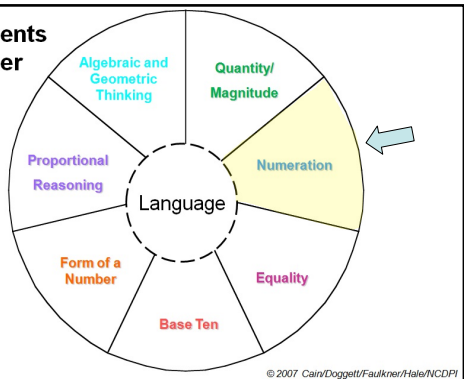
Translate this Relational Sentence…

There are 8 times as many raccoons as deer

Problem Solving Component



Components of Number Sense



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Numeration

Defining the Concept

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Numeration: Defined

numeration (noun) the action or process of calculating or assigning a number to something. a method or process of numbering, counting, or computing. ORIGIN late Middle English : from Latin ***numeratio(n) payment*** (in late Latin ***numbering***), from the verb ***numerare to number.***

Numeration Concepts

- Set – collection of elements
- Cardinal Number – identifies number of set
- Numeral – symbol representing number
- Rote counting – sequence of numbers
- Rational counting – one-to-one correspondence
- Ordinal numbers – first, second, etc.
- Manipulatives- concrete objects
- Skip counting – 2, 4, 6, 8

Skill Hierarchy

- SCOS (NCDPI)
- Common Core
- http://math.ncwiseowl.org/

Diagnosis

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Indicators of Issues:

- Rote Counting
  - Inability to count numbers accurately and sequentially
- Number Identification
  - Inability to name numbers rapidly
- Language
  - (The child has issues talking about numeration.)

    (Arithmetic on IQ test IQ/ Verbal side)

Where the research meets the road

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Introduction of Skills

Rote counting-

1. Determine how high they can count.
2. Stop student as soon as you hear a mistake.
3. Spread out practice.
   (10 min. is too long)

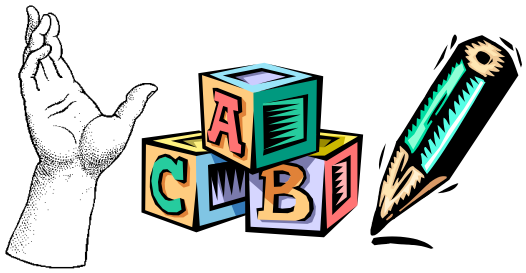
Introduction of Skills

Skip counting

1. Pre-skills
2. Rote count ten numbers for each series
3. Order of skills - counting by 10's, 2,5, 9, 4, 25,3,8,7,6
4. Model-lead-test
5. Add new numbers each day with at least two of the old numbers
6. Practice a number set

Classroom Application:

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application



Set 2 WCPSS Middle School Remedial Warm-ups

Wake County Public School System/Valerie Faulkner

Numeration across the NCSCOS

- Percents
- Ratios
- Decimals
- Fractions

10% of 100 **10% of 200**
10% of 50 **10% of 150**
10% of 23.28

a) What is happening here? The numbers are _____.

b) When you figure 10% of something, will your result be more or less than what you began with? _____

c) Is 10% more than ½ or less than ½ of something?_____.

d) If a basketball player made 10% of their foul shots, would that be very good, or not good at all? _____. Why? _____.

e) If a store item is on sale for 10% off, what does that mean? _____.

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Give 10% of the following numbers:

100→ _____ **400→ _____**

50→ _____ **800→ _____**

950→ _____ **1200→ _____**

135→ _____ **5→ _____**

46.5→ _____ **3.5→ _____**

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Find 10% of 800 → _____

Find 20% of 800→ **(10% + 10%)** _____

Find 30% of 800→ **(10% + 10% + 10%)** _____

Find 5% of 800→ **(1/2 of 10%)** _____

Find 15% of 800→ **(10% + 5%)** _____

Find 90% of 800→ **(100% - 10%)** _____

Find 110% of 800→ **(100% + 10%)** _____

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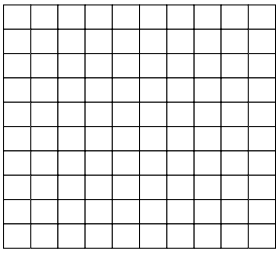
Your New Car!

You are buying a new car that is on sale for $27,000.

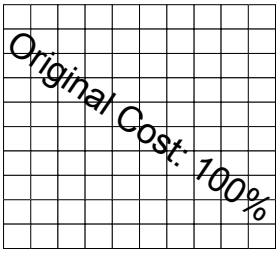
This is 80% of the Original cost of the car.

What was the Original cost of the car?

Using Hundreds Board to Solve Relatively Difficult Problems



Using Hundreds Board to Solve Relatively Difficult Problems



Using Hundreds Board to Solve Relatively Difficult Problems

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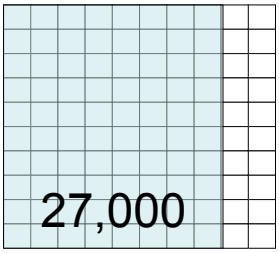
Using Hundreds Board to Solve Relatively Difficult Problems

How much
Is each 10th
of the whole?



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Using Hundreds Board to Solve Relatively Difficult Problems

How much
Is each 10th
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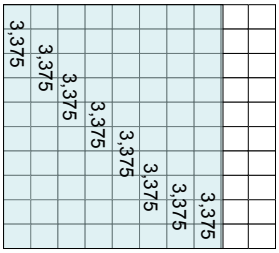

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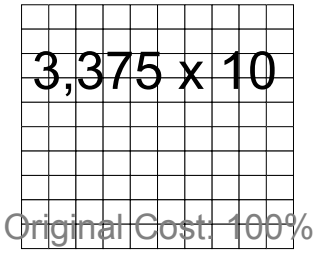
Using Hundreds Board to Solve Relatively Difficult Problems



The Essence of Percentages

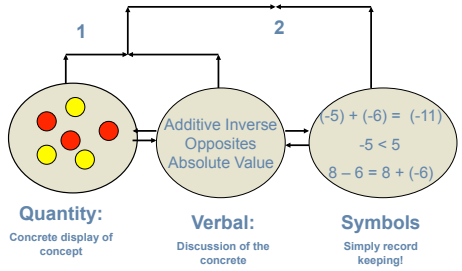
- It is a rate, not a raw number
- If you know 10%, you can find out anything!

What about Integers?

Tricks We Use

- Same-Change-Switch
- Bad Guys vs. Good Guys
- Matching Socks
- Just Subtract The Smaller From The Larger And Keep The Sign Of The Larger
  - (- 8 + 3)

Prototype for Lesson Construction



V. Faulkner and DPI Task Force adapted from Griffin

Frayer Model for Linguistics

Definition (in own words)

Whole numbers and their opposites

Integers

Characteristics

- **Positive, negative and zero**

Examples

Thermometer (-1 degrees)

Stock Market (Down 2)

Elevation (45 feet high)

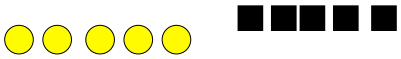
Non-Examples

0.56 $\sqrt{2}$ $\frac{1}{2}$

Concrete Display Of Integers

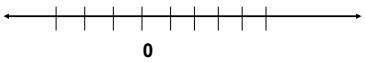
- Two Color Chips
- Number Line
- Four Pan Balance
- Thermometer

- Let black/yellow represent positive integers



- Let white/red represent negative integers.

Number Lines



Discussion Of Integers

- Elevation
- Stock Market
- Gains/Losses
- Temperature
- Golf
- Net Worth
- Positive and Negative Charges

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Retrieved from Weatherunderground.com 8/29/2011

Definition Of Math Fluency

Students are considered fluent in math if they are efficient, accurate, and flexible when working with math.

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References

- Hegarty, Mayer, Green, 1992
- NCDPI (http://math.ncwiseowl.org/)
- *V. Faulkner and DPI Task Force adapted from Griffin*
- *Stephan, Michelle L. , "What are Your Worth?" Mathematics Teaching in the Middle School, Volume 15, No. 1 August 2009.*

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Unit 6

REQUIRED READING

HANDOUTS

Student Math Profile Checklist—Special Ed Services

Does the student show evidence of the following?
YES—attach evidence
NO—attach evidence and describe techniques used to develop skill set
NT—Not Taught or no evidence of instruction on this skill—attach plan for instruction.

Base-ten Concepts, Operations on Numbers and their Meaning and Relationships:

____ Can count to ___10 ___12 ___20 ___50 ___100 ___1,000 ___∞
____ Knows more and less when difference is clear
____ Can use counting/one-to-one correspondence to solve problems (which one is more?--when difference is not clear)
____ Understands and can demonstrate an exchange of 10 ones units for 1 ten unit.

____ Can put ones units into a base ten form with manipulatives (13 ones→ exchange for 1 ten and model as 1 ten and 3 ones)
____ Can demonstrate that numbers can be put in different but equal forms
(27→ 27 ones, or 2 tens and 7 ones, or 1 ten and 17 ones or 13 ones and 14 ones, etc).
____ Can explain standard addition algorithm in terms of equal exchange of
___ones and tens
___ones, tens and hundreds
____ Can explain standard subtraction algorithm in terms of equal exchange of
___ones and tens
___ones, tens and hundreds
____ Understands and can demonstrate that subtraction is the inverse of addition

____ Can consistently derive correct answers within 20 with manipulatives/drawings, fingers, chips, hash marks, etc.

____ Can add/subtract ten with tens rods (ex: 22 + 10 = 32)

____ Can add/subtract other multiples of ten with ten rods (74-40 = 34)

____ Can add/subtract ten in head (ex: 22 + 10 = 32)
____ Can add/subtract other multiples of ten in head (74-40 = 34)

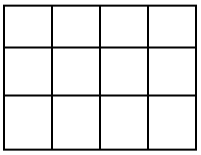
____ Can answer addition and subtraction facts within 20 with automaticity

____ Facts that are difficult/not automatic:

____ Understands and can demonstrate that multiplication is repeated addition
(3 x 4 = 3 + 3 + 3 + 3 = 12 → 3 carrots x 4 people means each person gets 3 carrots)

____ Understands and can demonstrate that multiplication can be shown through area and a rectangle:

3 linear in. x 4 linear in. = 12 1x1 squares



____ Uses patterns to access 1x1-digit multiplication

____ Uses "facts I know" to access other facts

____ Understands and can demonstrate that division is repeated subtraction

____ Understands and can demonstrate that division is the inverse of multiplication

____ Can solve 2 digit/1 digit division problems using multiplication

____ Can explain that regrouping in standard multiplication algorithm is regrouping tens

____ Can show the distributive property and use to multiply 1 x 2 digit numbers (8 x

____ Can explain that

$$\begin{array}{r} 45 \\ \underline{\times\ 37} \end{array}$$

represents
5 ones x 7 ones plus 4 tens x 7 ones plus 7 ones x 4 tens plus 4 tens x 3 tens

____ Other Strategies student uses to derive "Math Facts" with some fluency:

____ Can answer multiplication and division facts within 100 with automaticity

____ Facts that are difficult/not automatic:

See SCoS **Number and Operations Strand** Obj. 1.01 Grades K-3

Supplemental Program Recommendations to be delivered in a systematic and explicit manner over period of time:
Number Worlds—Development of early number concepts
Strategic Math Series—Base-ten and Concrete/Representational/Symbolic approach to operations
See also SpEd Instruction Checklist for further recommendations/information

Language and Math Translation:

____ Ability to translate language into math sentences

____Student recognizes addition problems and can

___Draw or demonstrate problem

___ Translate into abstract math sentence.

____Student uses "types of subtraction problems" to think through if a problem is addition or subtraction and to

___Draw or demonstrate problem

___ Translate into abstract math sentence.

____ Understanding of the concept of Equality on concrete level

____Student can demonstrate the concept of equality with their body as a scale

____Responds to equal quantities with even "scale"

____Responds to unequal quantities by lowering larger quantity

____ Understanding of the concept of Equality on symbolic level

____Student can discuss what an equal sign means in a math problem

____Student can explain that any two things that are equal can be put into a math sentence with an equal sign.

____Student can __add __subtract __multiply __divide same amount on both sides of equation WITHOUT variables and discuss what has happened (values have changed but equality has been maintained.)

____Student can __add __subtract __multiply __divide same amount on both sides of equation WITH variables and discuss what has happened (values have changed but equality has been maintained.)

____Student can set up an equality given several values and choosing two (Ex: given the values 10, 12, 6 + 5, 5 + 5, student can select the two equal terms and write them as an equality.

____Student can set up an equality given several more complicated values and choosing two (Ex: given more complicated values with words: The number of cards in a deck, a dozen, the number of sides in a triangle x 3, The number of days in a year – 100), the student can select the two equal values and write them as an equality.

____Ability to translate relational sentences into math sentences

____ Student uses strategy of figuring out "which one has more so how do we equalize" when translating problems. (Ex: *There are 5 dogs to every cat in the animal shelter this week.* Set up an equation. Think: There are more dogs so I'll need **to increase the amount of cats** to make the two amounts **equal**. I'll need 5 times more cats. D = 5C. Check: If there were 8 cats that would mean D = 5*8. 40 dogs. Yes, that makes sense, because I knew there had to be more dogs to start with (also show the non-example 5*D = C: If there were 8 dogs we'd have 40 cats, that doesn't make sense, because we know that we started out with more dogs.) Review: If there are 8 cats there are 40 dogs. In order to have the same amount of each type of animal, we needed to multiply the cats by 5.

See SCoS **Algebra Strand**: Grade 2—5.02; Grade 3—5.03, 5.04; Grade 4—5.02; Grade 5—5.02

* See Richard E. Mayer, "Mathematical Problem Solving" from Mathematical Cognition, Royer; Information Age Publishing

See also SpEd Instruction Checklist for further recommendations/information

Fractions:

_____ Knows that ½ of something is a fractional part.

_____ Understands that "half" means ½.

_____ Understands that ½ means precisely ½ (and not just a word for part of the whole).

_____ Has worked with fractions in concrete form and understands that 1/8 is smaller than ¼, etc.

_____ Has worked with fractions in concrete form and can explain and demonstrate why 1/16 is smaller than 1/8, etc. (it takes more pieces to make the whole so each piece is smaller).

____ Can explain what the numerator and denominator represent.

____ Can use concrete fraction pieces to compare fractions and figure out which represents a smaller or larger fractional part (3/8 vs. 5/16, etc.).

____ Can make exchanges with concrete fractions in order to simplify and combine fractions (1/8 **+ (¼)** = 1/8 **+ (1/8 + 1/8)** = 3/8).

____ Can make exchanges with abstract fractions by multiplying <u>by some form of 1</u> in order to simplify and combine fractions (1/8 + ¼ = 1/8 + ¼ (2/2) = 1/8 + 2/8 = 3/8).

____ Can explain why we are not changing the value of the fraction when we multiply by some form of 1 (identity principle) and that we are just breaking it into more pieces according to what we need).

____ Can demonstrate the concept of multiplying by a fraction by ripping up a piece of paper into fractional parts: 1 x ½ = ½ ½ x ½ = ¼ ¼ x ½ = 1/8

____ Can demonstrate the concept of multiplying by a fraction by repeated addition (3 x ¼ cup equals ¼c + ¼c + ¼c = ¾ c)

____ Demonstrates awareness that multiplying by whole numbers increases a value, but multiplying by fractions (less than one) decreases a value.

____ Can explain this difference (if I have 2 people and give them each 3 cups of milk, I gave out 6 cups total, but if I have 2 people and give them each ½ cup of milk I gave out less than 2, or 1 cup total).

___ Can multiply abstract fractions and explain if answer is sensible (¼ x 1/3 = 1/12—that makes sense because I do expect the answer to be a part of a fraction and smaller than ¼ or 1/3)

___ Can demonstrate what it means to show equal forms of a fraction--mixed number versus improper fraction (1 ¼ = → = → 5/4)

___ Can convert between abstract improper fractions and mixed numbers and explain that you are breaking up the wholes into equal fractional parts or grouping the fractional parts into equal wholes to express the same value in a different form.

___ Can demonstrate the concept of dividing by a fraction by using repeated subtraction (3 cups ÷ ¼ means you subtract out ¼c repeatedly→ you will end up with **<u>12</u>** portions of ¼ cup each).

___ Can explain why dividing by a whole number increases a value but dividing by a fraction increases a value (6 ÷ 2 means you have 6 cups and subtract out, or dole out, 2 cup servings until you are done which yields 3 servings; 6 ÷1/2 means you have 6 cups and dole out ½ cup servings, so you'll get more out of your 6 and end up with 12 servings).

___ Can divide abstract fractions and explain if answer is sensible (1/2 ÷1/4 = 2; that makes sense because I can get 2 servings of ¼ cup out of ½ cup.)

See SCoS **Number and Operations Strands**: Grade 2—1.02; Grade 3—1.05a, b, c, e; Grade 4—1.03; Grade 5—1.02; Grade 6—1.04b, d; Grade 7—1.02b

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Supplemental Program Recommendations to be delivered in a systematic and explicit manner over period of time:
Marilyn Burns Fraction Kit—Development of physical, conceptual and abstract understanding of fraction basics.
Lesson for Extending Fractions—Development of early work with operations on fractions.
Teaching Arithmetic: Lessons in Multiplying and Dividing Fractions—Operations developed through concrete understanding.
See also <u>SpEd Instruction Checklist</u> for further recommendations/information

Sped Math Instruction checklist

Elementary school

Indicate dates of instructional delivery and days per week.

- ❑ Instruction that followed the County Instructional Calendar
    - Duration of Implementation _____Days per week_____
    - Notes:

- ❑ Remedial instruction that uses a C-R-A approach
    - Strategic Math Series—Units:_____
        - Duration of Implementation _____Days per week_____
        - Notes:
    - Mathline
        - Duration of Implementation _____Days per week_____
        - Notes:
    - Touch Math
        - Duration of Implementation _____Days per week_____
        - Notes:

- ❑ Research-backed Remediation Program
    - Number Worlds: Level _____ Unit_____
        - Duration of Implementation _____Days per week_____
        - Notes:

Sped Math Instruction checklist

Middle school

Indicate dates of instructional delivery and days per week.

- Instruction that followed the County Instructional Calendar
    - Duration of Implementation _____Days per week_____
    - Notes:

- Remedial instruction that uses a C-R-A approach
    - Wake County Remedial Warm-ups
        - Level 1 ___ Level 2 ____
        - Duration of Implementation _____Days per week_____
        - Notes:
    - Strategic Math Series—Units:_____
        - Duration of Implementation _____Days per week_____
        - Notes:

- Research-backed Remediation Program
    - Number Worlds: Level _____ Unit_____

        - Duration of Implementation _____Days per week_____

        Notes:

POWERPOINTS

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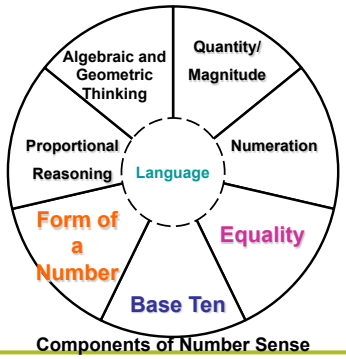
Unit 6
Base Ten, Equality and Form of a Number



Components of Number Sense

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Equality

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Equality is a mathematical statement of equivalence of two quantities and nothing more.

Cain, Faulkner, Hale 2007

Base Ten

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Base Ten-Defined

- Base ten digits are often used with a decimal separator and includes the start of a fractional point and positive and negative numbers.
- Linguistically, when we use English, the structure of the number words shows a base of ten, at least at the outset. When we write numbers, the structure of our number symbols also shows base ten.
- Sign language mimics our language not our number system.

Walter S. Sizer Base and Subbase in a Number System

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Form of a Number

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Form of a number

Form of the number can be defined as multiple representations of quantity, ratios, and mathematical information.

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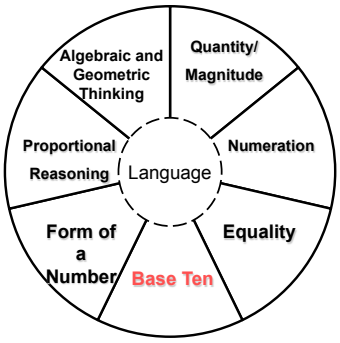

Components of Number Sense

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Diagnosis

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Base Ten

- Has the student been exposed to base ten concepts?
- What are the early signs of base ten issues?
- Math Student Profile Checklist

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Where Research Meets the Road

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Sharon Griffin's developmental nature of children using two number lines.

- 6 year old structure
- 8 year old structure
- 10 year old structure

Piaget's developmental nature of learning

- Motor (0-2)
- Preoperational (2-7
- Concrete (7-11)
- Formal (11-Adult)

Vygotsky - Zone of proximal development

- Constructivist
- We can move students through Piaget's stages more quickly depending on the types of activities in which engage them.
- Learning happens just above the mastery level. (ZPD)

Classroom Application:

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Base Ten

- Sharon Griffin
- Liping Ma
- John Woodward

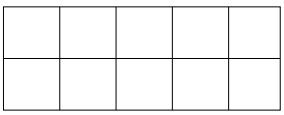
Math Facts

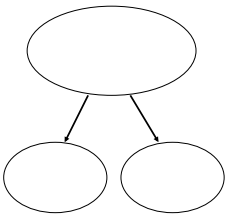
8 + 5
Fact Families?
Flash Cards?
Rewards?

Problem?
We see this as a one-step building block.

Cognitively, it is not.

Making 10: Facts within 20





Makes 10 Left over

_____ + _____ = _____

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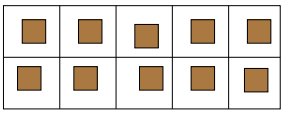
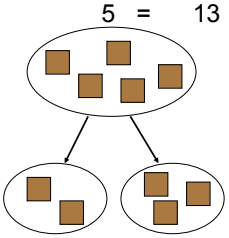
Making 10: Facts within 20

8 + 5 = 13





Associative Property of Addition

__8__ + __(2 + 3)__ =

Makes 10 Left over

(8 + 2) + 3=

(10) + 3= 13

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Base Ten

My student can't compute 42 plus 10. How do you remediate this student?

My student subtracts 108 – 19 and arrives at an answer of 89, but cannot explain their answer. Does that tell you that they understand base ten? How would you know?

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Base Ten

- My student is multiplying 20 x 3 and writing the entire problem out and solving it procedurally and getting it correct. How do you respond?
- My student divides 108 by 9 and arrives at 2 for the quotient? How do you respond?

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Base Ten

- You have a student who struggles with memory and clearly cannot do 2-digit by 3-digit multiplication. She is a seventh grader and must be able to perform computations with decimals, fractions and percent.
- How do we help her to use base ten to allow her access these SCOS skills?

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Transitional Math
John Woodward's Strategies

Building Number Sense

John Woodward Resources

Addition Number Sense
Base Ten

43 + 12 → 40 + 3 + 10 + 2

40 + 10 + 3 + 2

50 + 5

Answer: 55

53
+ 15

50	3
10	5
60	8 → 68

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Addition with Regrouping

29
+ 15

20	9
10	5
	14
30	10 + 4 → 44

10

20	9
10	5
40	4 → 44

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Try It!

49 + 37 →

Decompose both numbers into tens and ones

Combine ones

Trade group of ten ones for 1 ten.

Combine tens.

Answer →

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Subtraction

76	70	6
- 29	- 20	9
	60 + 10	6
	- 20	9
	60	6+10 (16)
	- 20	9
	40	7

Answer: 47
Estimate: 80 – 30
Calculator Check: 47

You Try It!

81		
- 52		

Estimate:
Calculator:
Answer:

95 →	90	5
x 3	x	3
		15
		270
		285

Try It!

83 x 7

x _____

_____

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What about Larger Numbers?

- **Sense making of the numbers.**
- **In the real world we use a calculator to at least check our work.**

Use Estimation to get a ballpark number.

389 x 78 ~ 400 x 80 =

4 x 100 x 8 x 10=

4 x 8 x 100 x 10= 32 x 1000=32,000

297 x 31 ~ 300 x 30 = 9000

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Division

- Use a Number Line
- Use Extended Facts
- Use Estimation

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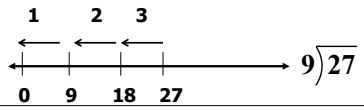
Division
(Works with facts and Conceptually Guided Operations.)
Use of the Number line



$9\overline{)27}$

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Division

- Estimation
  - Promotes number sense
  - Gives students an explicit strategy to check solutions to problems they work on a calculator.
  - Uses the strategy of near fact.
  - Helps with two digit multiplication by removing power of tens.

$3\overline{)48}$

$\begin{array}{r} 10 \\ 5\overline{)50} \end{array}$

$34\overline{)239}$

$30\overline{)240}$

$3x10\overline{)24x10}$

$\begin{array}{r} 8 \\ 3\overline{)24} \end{array}$

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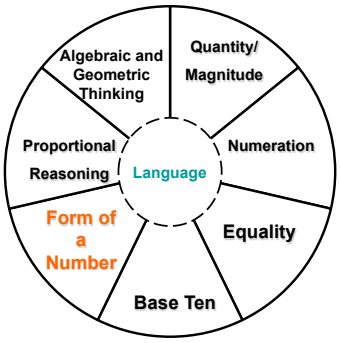

Components of Number Sense

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Diagnosis

Equality/Forms of a Number

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Diagnosis Strategies

- What kinds of experiences have students had with understanding and exploring equality/forms of a number?
- Can the student tell you what equality is?
- Can they recognize different forms of a number?
- Does the student have an internal sense of a balance scale?
- Can students explain what it means that these representations are different forms of equal values?
  - Example .45 = 45/100 = 9/20=45%

Assessments
Number Knowledge Test
Informal probes (Craven County Probes)

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Where Research Meets the Road

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Research Equality / Form of a Number

- What does the research say about using equality and form of a number
- TIMSS (1999, 2003)
- Ball, 2006
- Ma, 1999

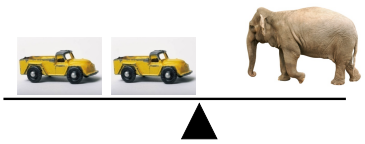
Classroom Application:

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Are these equal?

$$1 = 1^3$$

Are these the same?



Are two cars equal to two cars?
Is one elephant equal to two cars?
Are these the same?

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What about this?

X = Y on a balance scale

3x + Y = 0 on balance scale

What else could you put on the left hand side the balance?

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- Discuss in small groups base ten concepts for addition, subtraction, multiplication, division, decimals and percent.
  - Assignments by tables of concept
  - Report out by group
  - Now try it!
    - Change the way we talk to kids
    - Role playing with student mistakes.

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Addition

1) 16 + 7 = 113

2) 14 + 3 = 17

3) 35 + 81 = 116

4) 62 + 8 = 610

5) 407 + 63 = 4610

6) 569 + 724 = 12813

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Subtraction

34 - 2 = 212

86 - 7 = 79

71 - 69 = 2

42 - 27 = 15

56 - 51 = 115

854 - 60 = 7814

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Multiplication

1) 17 x 5 = 205

2) 40 x 8 = 320

3) 23 x 4 = 122

4) 27 x 31 = 27, 121 = 1237

5) 54 x 19 = 726, 54 = 1266

6) 56 x 28 = 728, 122 = 1948

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Division

1) $2\overline{)86}$ = 34; 8; 6; 6

2) $4\overline{)20}$ = 5; 20

3) $7\overline{)133}$ = 91; 7; 63; 63

4) $3\overline{)619}$ = 62; 6; 19; 18; 1

5) $8\overline{)816}$ = 201; 8; 16; 16

6) $6\overline{)678}$ = 311; 6; 7; 6; 18; 18

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Decimals

1) 24.3 + .59 = 30.2

2) 6.7 + .88 = 15.5

3) 4.52 + .078 = 5.30

4) 379.432 + 23. 556 = 61.4992

5) 72.34 + .6672 = 1.3906

6) 8.216 + .797 = 16.186

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It's All About the Story

- You say 13-5, and the student responds by placing 13 ones on the base ten mat and then takes away 5. How should the teacher respond?
- A student says that 0.5 + 0.3 and 0.4 + 0.4 are the same problem. How do you respond?
- A student says 3 – 2 is the same as 3 + -2. How do you show the student that these statements are not the same, but they have the same result?
- I asked the student if 0.50 and 50% is the same thing and they say no. How do you respond?

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References

- Ball, 2006
- Ma, 1999
- John Woodward Transitional Math
- TIMSS (1999, 2003)

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Assignments from Units 5 and 6

- **Learning Task 5 (See Rubric)**
  - **Structures of Addition, Subtraction, Multiplication and Division**
- **Learning Task 6 (See Rubric)**
  - **Number Knowledge Test with Reflection**

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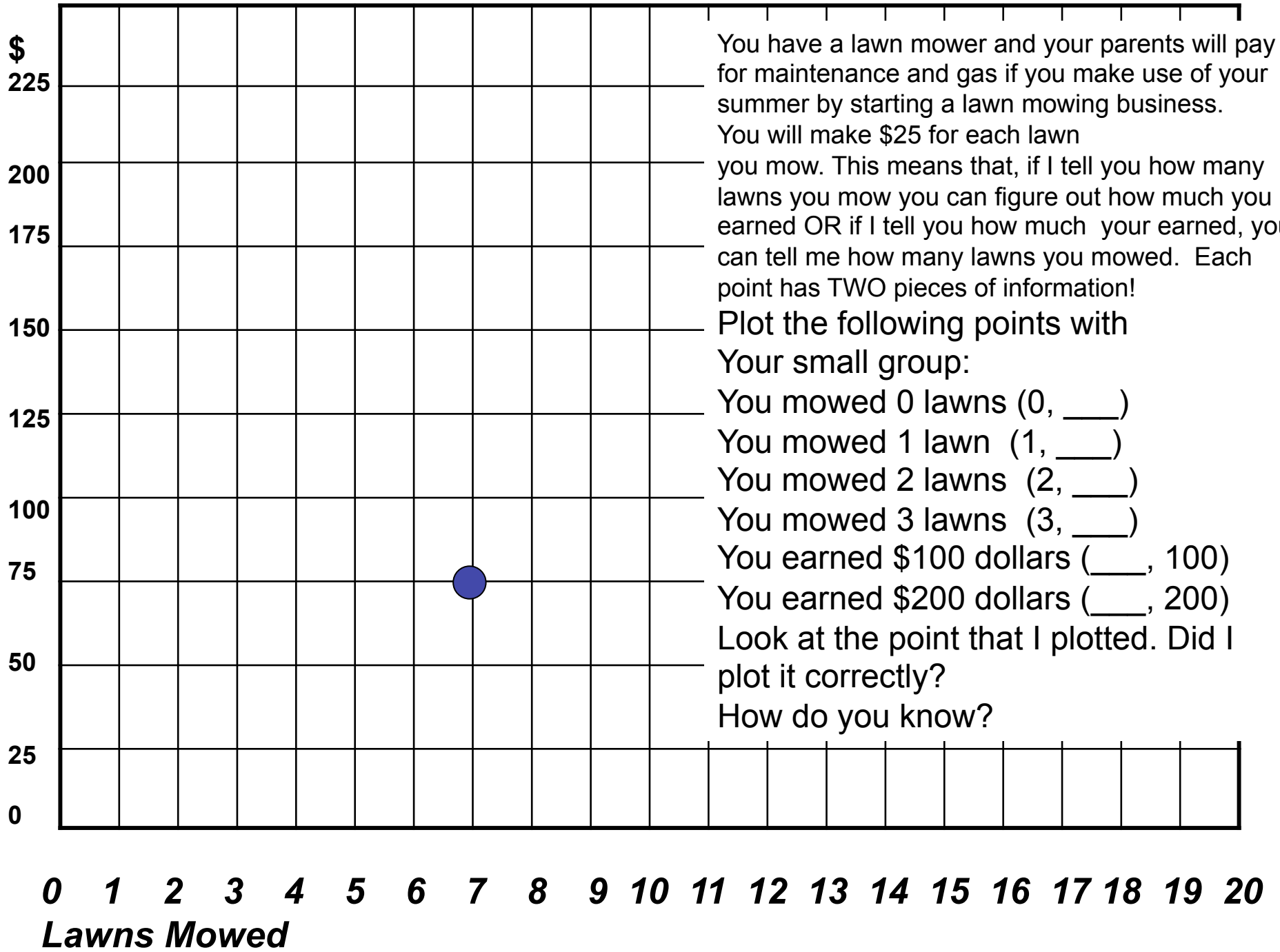
Unit 7

REQUIRED READING

HANDOUTS



You have a lawn mower and your parents will pay for maintenance and gas if you make use of your summer by starting a lawn mowing business. You will make $25 for each lawn you mow. This means that, if I tell you how many lawns you mow you can figure out how much you earned OR if I tell you how much your earned, you can tell me how many lawns you mowed. Each point has TWO pieces of information!

Plot the following points with Your small group:

You mowed 0 lawns (0, ___)

You mowed 1 lawn (1, ___)

You mowed 2 lawns (2, ___)

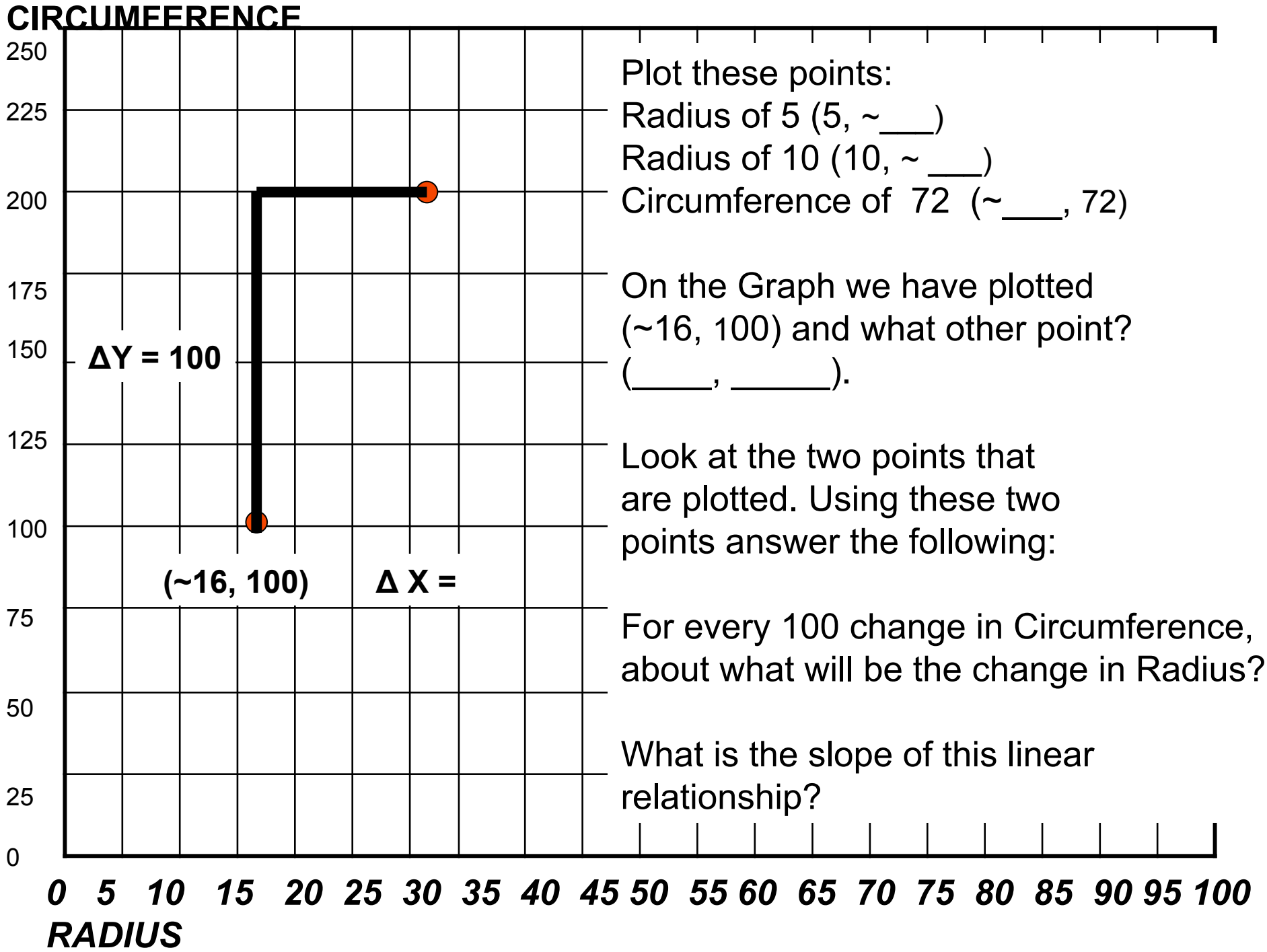
You mowed 3 lawns (3, ___)

You earned $100 dollars (___, 100)

You earned $200 dollars (___, 200)

Look at the point that I plotted. Did I plot it correctly?

How do you know?



Plot these points:
Radius of 5 (5, ~___)
Radius of 10 (10, ~ ___)
Circumference of 72 (~___, 72)

On the Graph we have plotted (~16, 100) and what other point? (____, _____).

Look at the two points that are plotted. Using these two points answer the following:

For every 100 change in Circumference, about what will be the change in Radius?

What is the slope of this linear relationship?

Survey for Unit 7

Answer all questions below (red, green and blue)

Red dots

Teaching experience in years _____

How many more years do you plan to teach_____

Green dots

Number of letters in first name _____

Number of first cousins _____

Blue dots

Distance you drove (in miles) to get here _____

Number of minutes it took you to get here _____

Two shoppers, One purpose

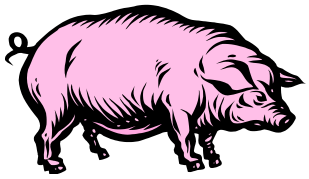
Jenn and Jermaine had a little argument about shopping. Jenn said that she could shop for things cheaper than Jermaine, because "guys don't know how to shop." Jermaine thought that was ridiculous and challenged her to a shopping match. "You're on" she said. "Great" said Jermaine, "Save your receipt and you need to include the cost of travel to get to the store." Jenn agreed to these conditions and they were ready to go.

Jenn had an energy efficient hybrid car, so she felt confident that she would win, because Jermaine drove a truck. Puggly Wuggly was far away, but she thought the cheap prices at Puggly's $2/item bag day sale would be worth the drive. She spent $2.00 on gas for the whole trip.

Jermaine knew about the Puggly Bag Sale too. But he didn't drive his truck. He took the bus and it cost him $1.50.

Model the contest between the two with linear equations and then graph them. Answer the questions and be prepared to discuss with the class.

Mathematical Models



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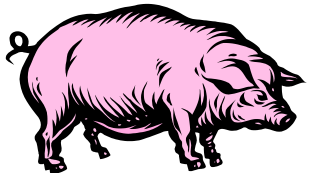

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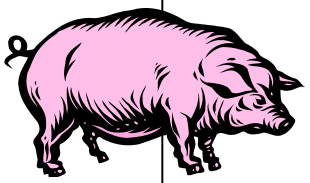
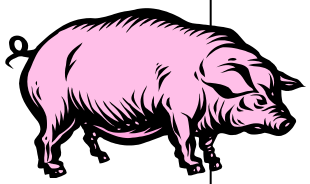
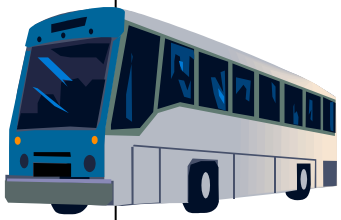
x Number of Items	Rule Math Model	y Cost
	+	

x Number of Items	Rule Math Model	y Cost
	+	

POWERPOINTS

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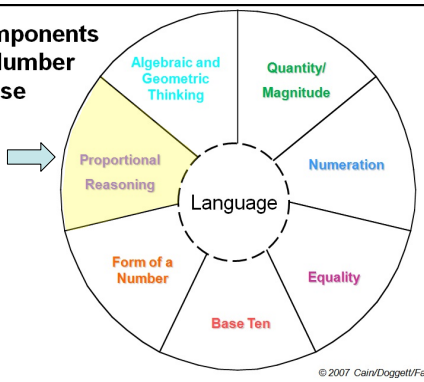
Unit 7
Proportional Reasoning
Algebraic and Geometric Thinking

Components of Number Sense



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Proportional Reasoning

Defining the Concept

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Proportional Reasoning

- Proportional reasoning involves a multiplicative relationship between two quantities.
- Proportional reasoning is one of the skills a child acquires when progressing from the stage of concrete operations to the stage of formal operations.

Key Ideas of Proportional Reasoning

- All ratios have a multiplicative relationship between values.
- When two ratios are equal, the multiplicative relationships "within each of the individual ratios are the same"

Scalar Method (Within)---

- Uses the knowledge that the multiplicative relationships are the same within each ratio in a proportion to solve for a missing value

$$2 : 10 = 4 : x$$

Functional Method (Between)
AKA 'Scale Factor'

- Multiplicative relationships "between" the two ratios in a proportion also exist.

2:10 = 4:20

- If we multiply both numbers in the ratio 2:10 by 2, we obtain the values in the second ratio. (Remember we are really multiplying our value by a form of one.)

$$\frac{2}{10} \times \frac{2}{2} = \frac{4}{20}$$

Language:

- Within (Scalar)
- **Relationship within a figure or ratio**
- We use the Within multiplicative relationship as our factor to maintain PROPORTIONALITY and/or SIMILARITY

- Between (Functional)
- **Scale Factor – How many times larger or smaller? What is our Scale?**
- We use the Between multiplicative relationship as our **Scale Factor** to move between different SCALES

Why does this work?

If Sam can mow one lawn in 2 hours, how many lawns can he mow in 8 hours?

Write a proportional relationship that represents the situation.

$$\frac{2}{8} = \frac{1}{x} \qquad \frac{1}{x} = \frac{2}{8} \qquad \frac{2}{1} = \frac{8}{x}$$

$$\frac{x}{1} = \frac{8}{2} \qquad \frac{1}{2} = \frac{x}{8} \qquad \frac{8}{x} = \frac{2}{1}$$

Write a non-example.

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Diagnosis

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Proportional Reasoning---

- Is foundational to higher mathematics
- Does not always go hand-in-hand with symbolic representation
- Has four levels of solution strategies
  - 0 (use of additive strategies, solution arrived at by luck)
  - 1 (use of pictures, models, manipulatives)
  - 2 (use of level 1 strategies and multiplication and division strategies as well)
  - 3 (use of cross-multiplication or equal ratios)

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Student Issues

- Students do not analyze the relationships among symbols.
- Students use the cross-product method even when the within or between relationships are obvious.

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Where the research meets the road

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

"We view proportional reasoning as a pivotal concept. On the one hand, it is the capstone of children's elementary school arithmetic; on the other hand, it is the cornerstone of all that is to follow." (Lesh, R., Post, T., & Behr, M. (1988)

Four Major Issues

- Emphasis on procedures vs. developing sense of the meaning of rational numbers
- We teach as if Level 3 were the access point, rather than Level 1
- Use of representations in situations where rational numbers and whole numbers are easily confused.
- The inherently confusing nature of fractional notation to represent proportions
  - (1 out of 6 versus 1/6)

Joan Moss and Robbie Case 1999

The inherently confusing nature of fractional notation to represent proportions

Joan Moss and Robbie Case 1999

1/6 or fraction of stable unit like money (9/20) (Story One)

VERSUS

1 out of 6 or proportion of a changing unit (8/30)(Story Two)

How Do We Represent the Money?

How do we determine this ratio?

_45_ /_100__

1 Dime

7 Nickels

How Did the Team Do Altogether?

How do we determine this ratio?

__8_ /_30_

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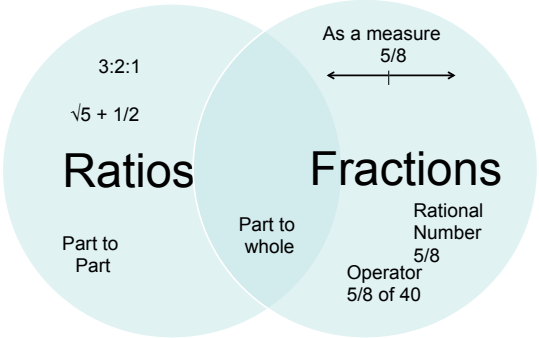

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FRACTIONS MATTER:
Teaching Fractions must include connections to Proportional Reasoning!

1/6 + 1/6 = 2/6

1/6 + 1/6 = 2/12

What is my Unit?
I can't make sense of a fraction unless I know what my comparative unit is!

How does my **portion** size affect my servings?

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Solutions

- Greater emphasis on meaning
- Greater emphasis on the proportional nature of fractions
- Greater emphasis on Level 1 and Level 2 solution strategies
- Greater development of diagram literacy

Joan Moss and Robbie Case 1999

Proportional Reasoning

- Develops hand in hand with Quantity and Magnitude.
- Is a skill the child obtains when moving from the concrete to the formal operational stage of development.
- Can relate equal lengths of unlike units.
- Can be conceptually understood through diagram literacy.

Classroom Application

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

4th Grade NAEP Item

- A fourth grade class needs five leaves each day to feed its two caterpillars.
- How many leaves would it take to feed twelve caterpillars?
- Solve this problem using Level 1 and Level 2 strategies

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Paper Clip Chains - AIMS Activity

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Helping the Process

- Strong instruction will help students to find the strategies that will involve the fewest computations.
- Students who feel comfortable with a variety of solution strategies will increase their conceptual understanding, as well as their procedural fluency with proportions.

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Explore Proportions through Similarity

- –Enlargements
- –Reductions
- –Scale factors
- –Indirect Measurement

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Students Need Opportunities to…

- Reason about proportional situations
- Find unit rates
- Construct equivalent ratios
- Link ratios, percent and fractions

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Bridging to Level 3

Requires a strong understanding of the components of proportional reasoning:

- – Change between equivalent ratios is multiplicative, not additive
- – The multiplicative change is constant
- – The relationship between ratios is the scale factor (If you multiply one ratio by the scale factor, the result is the second ratio.)

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What Component of Number Sense can help us explain why cross multiplication works?

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Problems that Encourage Proportional Sense

Adapted from Esther Billings
Mathematical Teaching in the Middle School NCTM

John drove 60 miles in 2 hours. If he continues to drive at this speed, how long will it take him to drive 40 additional miles?

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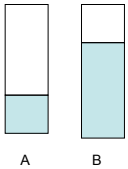
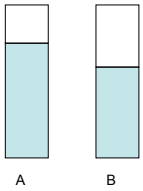
- In the following situations it is indicated which carafe contains the stronger coffee. Determine which carafe will contain the stronger coffee after the alterations have been made.
- Explain how you came to your answer.

Carafe B contains weaker coffee than Carafe A. Add one spoon of instant coffee to carafe A and one cup of water to carafe B.



Which carafe will have the stronger coffee?

Carafe A and Carafe B contain coffee that tastes the same. Add one spoon of instant coffee to both carafe A and B.

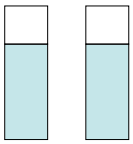


Which carafe will have the stronger coffee?

Trashketball and Carafes?

- Trashketball is a proportion too…
- How would you use student knowledge of Trashketball to support their understanding of the carafe situation?

Carafe A contains weaker coffee than Carafe B. Add one spoon of instant coffee to carafe A and carafe B.



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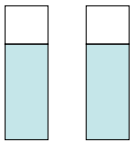


Which carafe will have the stronger coffee?

Exposing students to nonnumeric quantities will strengthen their proportional reasoning.

Why don't kids think proportionally?

- Because we don't build on their intuitive thinking.
- Because it's hard for them.
- Because we don't teach it very well.
- Because we don't make connections between different forms of numbers.
- Because it's hard for us…

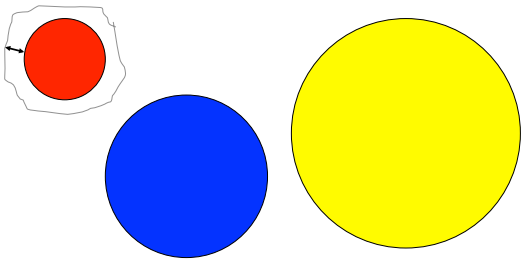
Perimeter around the earth

Imagine there is a string that goes around the earth and represents the earth's perimeter at it's widest arc.

Now increase that string by 1 meter.

How far away would the string now be from the earth's surface?

Let's use visuals to help us think this through…



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At Your Tables---

- Use the string to represent the circumference of your circle
- Add 1 meter (100 cm) of string to the circumference
- Cut your string. The length of your string represents the circumference of your circle plus 100 cm.
- Use your string to make a larger circle around your original circle. (Both circles should have the same center.)
- Measure and record the distance from the edge of the new circle to the edge of the original circle. (in cm)
- About how much of a meter does this amount represent?

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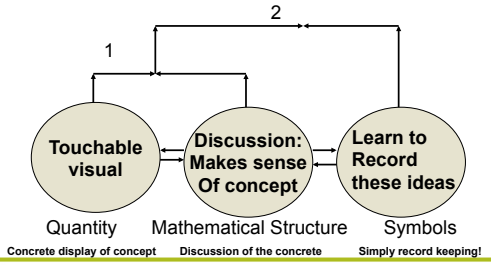
Prototype for lesson construction



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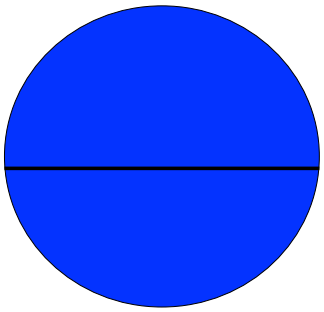
V. Faulkner and DPI Task Force adapted from Griffin



$$\frac{c}{d}$$

Circle Relationships

If given d
$d \bullet \pi = C$

If given r
$(2 \bullet r) \bullet \pi = C$

If given C
$C \div \pi = d$
$C \div \pi \bullet 2 = r$

Questioning Techniques: Developing Number Sense

If a circle has a circumference of 12, about what is it's diameter?

If a circle has a diameter of 10, about what is its circumference?

If a circle has a circumference of 32, about what is its diameter? It's radius?

If a circle has a radius of 4, about what is its circumference?

Proportional Thinking…

Let's reinforce what we know about proportional relationships to make sure we are ready to reason with our string around the earth problem…

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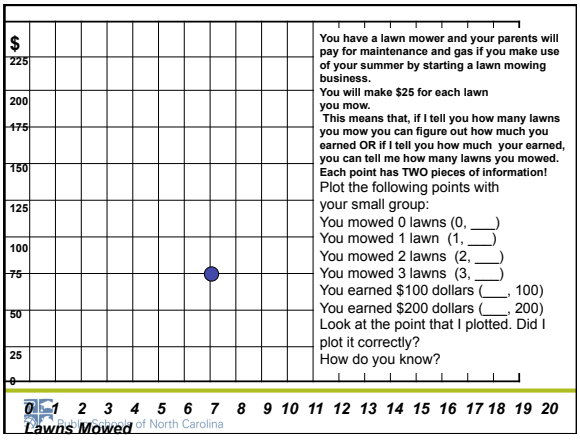

You have a lawn mower and your parents will pay for maintenance and gas if you make use of your summer by starting a lawn mowing business.
You will make $25 for each lawn you mow.
This means that, if I tell you how many lawns you mow you can figure out how much you earned OR if I tell you how much your earned, you can tell me how many lawns you mowed.
Each point has TWO pieces of information!

Plot the following points with your small group:
You mowed 0 lawns (0, ___)
You mowed 1 lawn (1, ___)
You mowed 2 lawns (2, ___)
You mowed 3 lawns (3, ___)
You earned $100 dollars (___, 100)
You earned $200 dollars (___, 200)
Look at the point that I plotted. Did I plot it correctly?
How do you know?

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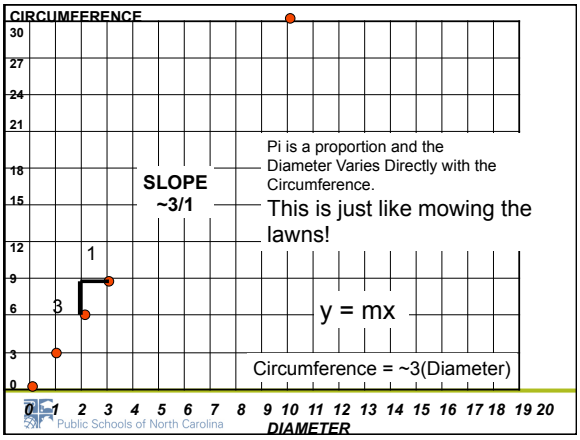
Now, back to the earth---


CIRCUMFERENCE
30
27
24
21
18
15
12
9
6
3
0
SLOPE
~3/1
1
3
Pi is a proportion and the Diameter Varies Directly with the Circumference.
This is just like mowing the lawns!
y = mx
Circumference = ~3(Diameter)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
DIAMETER
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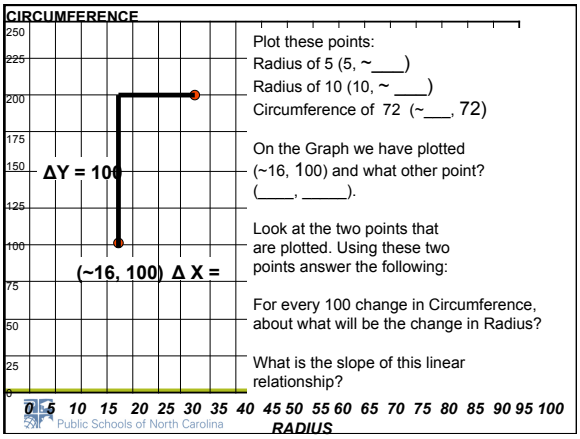
CIRCUMFERENCE
250
225
200
175
150
125
100
75
50
25
0
ΔY = 100
(~16, 100) Δ X =
Plot these points:
Radius of 5 (5, ~____)
Radius of 10 (10, ~ ____)
Circumference of 72 (~___, 72)
On the Graph we have plotted (~16, 100) and what other point? (____, _____).
Look at the two points that are plotted. Using these two points answer the following:
For every 100 change in Circumference, about what will be the change in Radius?
What is the slope of this linear relationship?
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
RADIUS
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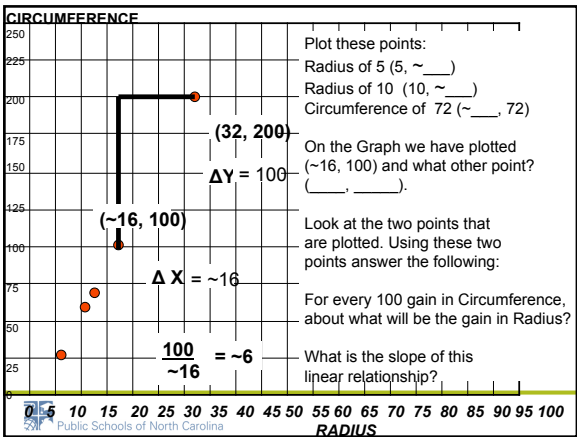
CIRCUMFERENCE
250
225
200
175
150
125
100
75
50
25
0
(32, 200)
ΔY = 100
(~16, 100)
Δ X = ~16
100
~16
= ~6
Plot these points:
Radius of 5 (5, ~___)
Radius of 10 (10, ~___)
Circumference of 72 (~___, 72)
On the Graph we have plotted (~16, 100) and what other point? (____, _____).
Look at the two points that are plotted. Using these two points answer the following:
For every 100 gain in Circumference, about what will be the gain in Radius?
What is the slope of this linear relationship?
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
RADIUS
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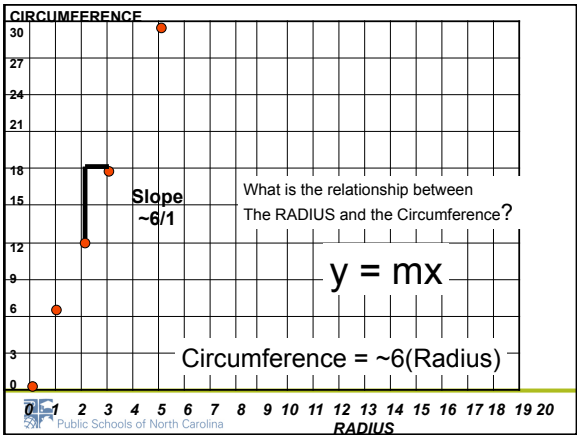
CIRCUMFERENCE
30
27
24
21
18
15
12
9
6
3
0
Slope
~6/1
What is the relationship between
The RADIUS and the Circumference?
y = mx
Circumference = ~6(Radius)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
RADIUS
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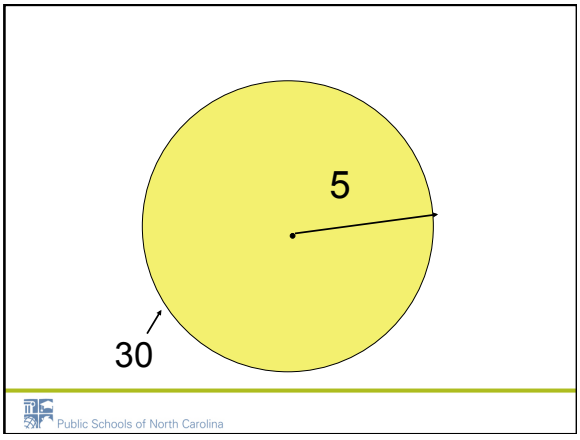
5
30
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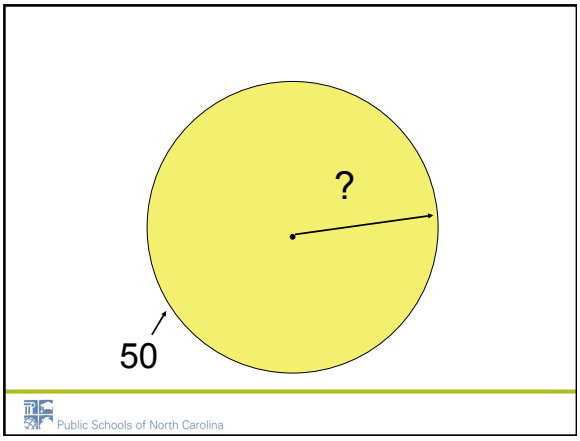
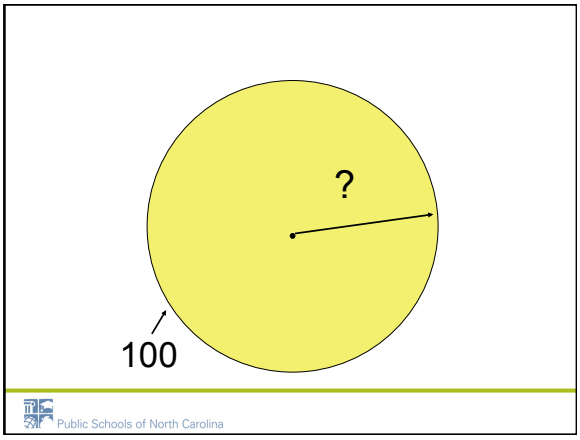
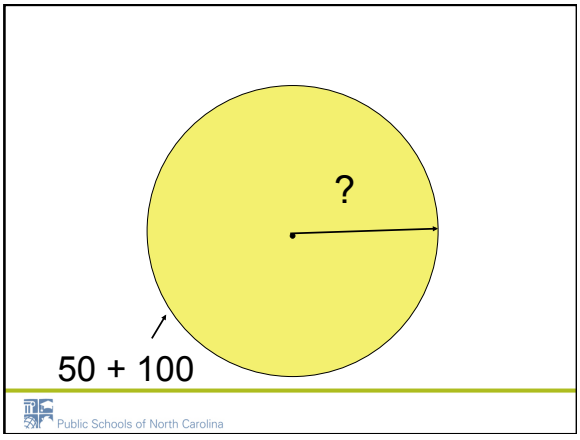
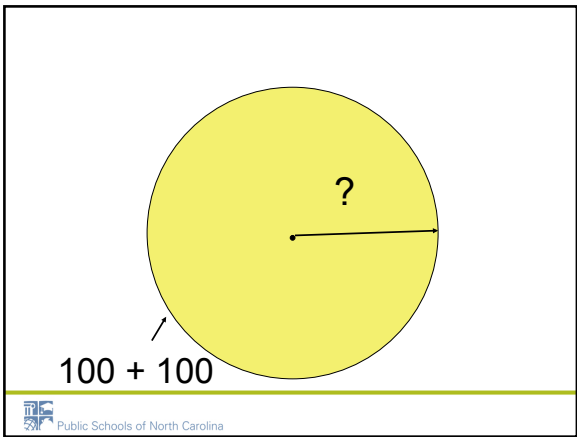
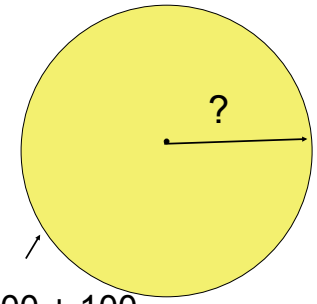

?
50
Public Schools of North Carolina


?
100
Public Schools of North Carolina


?
50 + 100
Public Schools of North Carolina


?
100 + 100
Public Schools of North Carolina



1,000,000 + 100

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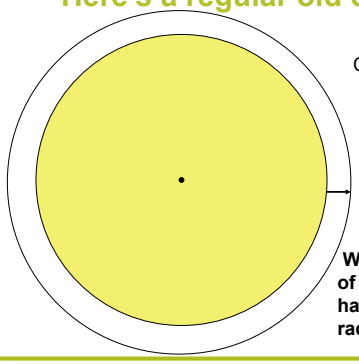
Here's a regular old circle…



Circumference is 50 cm

With our added meter of circumference we have 16 cm of added radius.

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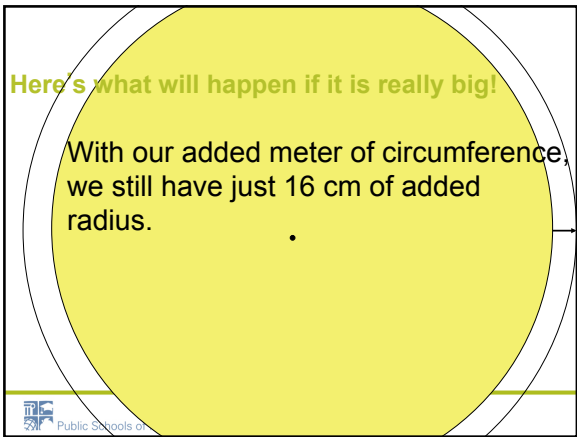
Here's what will happen if it is really big!

With our added meter of circumference, we still have just 16 cm of added radius.



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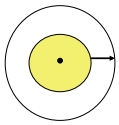
What if it gets really small?

Still, when we add a meter of circumference we still have just 16 cm of added radius.



And what if we engage in a 'thought experiment' ?

What if our circle goes down to a circumference of 0?

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What if we have a circumference of 0?

With, a circumference of 0 we'd have <u>no circle at all.</u>
We can still add our meter of string to nothing.
It will go around a point and we'd STILL be 16 cm away from our center, and have a radius of ~16cm

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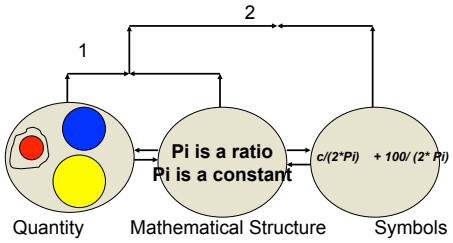
Pi and the earth



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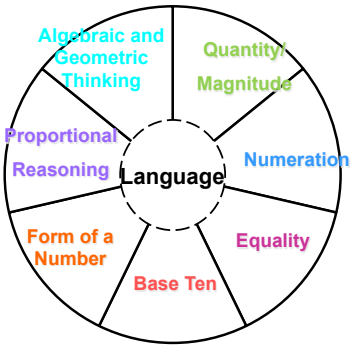

Components of Number Sense

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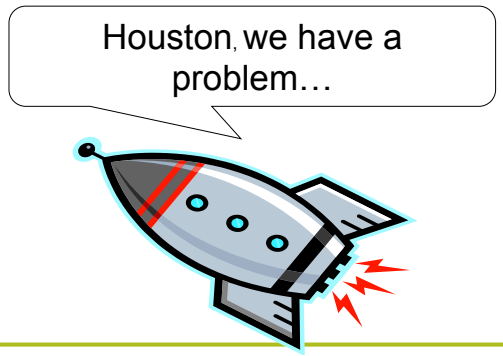

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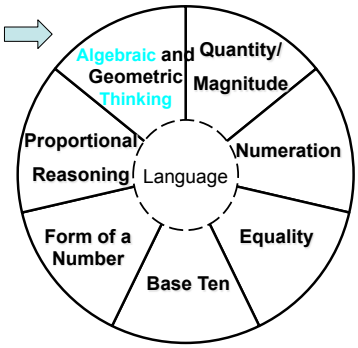
Algebraic and Geometric Thinking



The Components of Number Sense

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Algebraic Thinking

Defining the Concept

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Defining the Concept

The study of the letter x?

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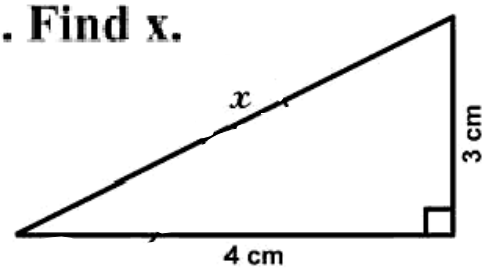
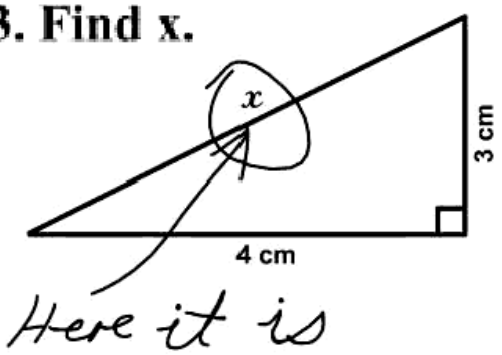
3. Find x.



3. Find x.



Think about algebra as---

a) a systematic way of expressing generality and abstraction
b) the syntactically guided transformation of symbols.

(Adding It Up, National Research Council)

In simpler terms, algebra is

- Using what we know to find out what we don't know.
- Analyzing patterns and relationships.
- Making predictions, modeling situations, and solving problems.

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Verbal:

You and your friends want to record a CD. The company you want to hire charges $300 for making the master and designing the art for the cover. It charges $2.50 for burning each CD. Use the function rule P(c)=300 + 2.5c. Make a table of values and a graph.

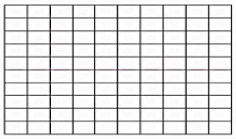
Algebraic

_____

Table

_____ (x)	Function: _____	_____ (y)

Graph



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Diagnosis

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

From Elementary School Arithmetic, a student brings---

- A desire to execute operations rather than use them to represent relationships.
- The use of an equal sign to announce a result rather than to show equality.
- A use of inverse or undoing operations to solve a problems.
- A perspective of letters as representing unknowns but not variables.

Challenges Students Experience with Algebra

- **Translate word problems into mathematical symbols**
- **Describe, paraphrase, or explain**
- **Link the concrete to a representational to the abstract**
- **Remember vocabulary and processes**
- **Show fluency with basic number operations**
- **Maintain focus**
- **Show written work**

At the Elementary Level, Students with Disabilities Have Difficulty with:

- **Solving problems** (Montague, 1997; Xin Yan & Jitendra, 1999)
- **Visually representing problems** (Montague, 2005)
- **Processing problem information** (Montague, 2005)
- **Memory** (Kroesbergen & Van Luit, 2003)
- **Self-monitoring** (Montague, 2005)

At the Middle School Level, Students with Disabilities Have Difficulty:

- **Meeting content standards and passing state assessments** (Thurlow, Albus, Spicuzza, & Thompson, 1998; Thurlow, Moen, & Wiley, 2005)
- **Mastering basic skills** (Algozzine, O'Shea, Crews, & Stoddard, 1987; Cawley, Baker-Kroczynski, & Urban, 1992)
- **Reasoning algebraically** (Maccini, McNaughton, & Ruhl, 1999)
- **Solving problems** (Hutchinson, 1993; Montague, Bos, & Doucette, 1991)

Where the Research Meets the Road

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

NAEP Task

Give the value of y when x = 3.

x	1	3	4	7	n
y	8		11	14	

3X +7 = Y

- Write a story problem for this equation…

How would your students do?

7th grade?
8th grade?
Algebra 1?
Algebra 2?
Calculus?

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Algebra is --

- A gateway to higher levels of mathematics education.
- Abstract thinking required for our full participation in our democratic society and technological driven world.

Dudley and Riley 1998

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"For students to meaningfully utilize algebra, it is essential that instruction focus on sense making, not symbol manipulation. Throughout their mathematical careers, students should have opportunities to reflect on and talk about general procedures performed on numbers and quantities."

Battista and Brown (1998)

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Concrete to Representational to Abstract

- Algebra is fundamentally very concrete!
- Like all math, Algebra is just a way to model concrete realities, actions and situations.
- Why do we bypass the concrete reality of algebra and go directly to symbolic manipulation?

Classroom Application

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Relating Quantities

- Teaching Experience,
  Number of Years you plan to continue to teach
(Red Dots)
- Number of letters in your first name,
  Number of First Cousins
(Green Dots)
- Distance in miles you drove,
  Time traveled in Minutes
(Blue Dots)

Relating Quantities

Children make natural connections between numbers.

Children need

- many opportunities to investigate relationships
- visuals provided as numerical data is compared
- non-examples to help set the limits of a concept and to fully structure their understanding.

What does the relationship mean?

- How is data related or not related?
- What new data might come in at a later time?
- How could you use the relationship to predict another data point?
- What could have happened if?
- What happens between data points?
- What is the essence of this pattern?

Other Examples

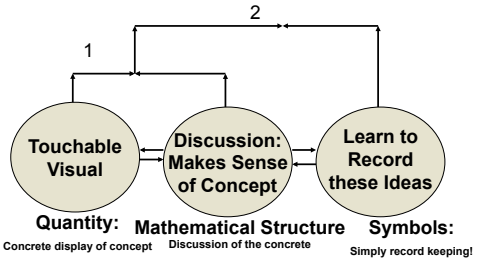
- Number of Pets , Number of family members
- Grade level, Average age
- Number of miles driven, gas in tank
- Age/Grade on Report Card in Math
- Length of foot, height
- Amount of trouble you get into, parent happiness factor

Ordered Pairs can have Predictive Value--

But they do not always represent a Cause and Effect

Example: Vocabulary, Height

Prototype for Lesson Construction



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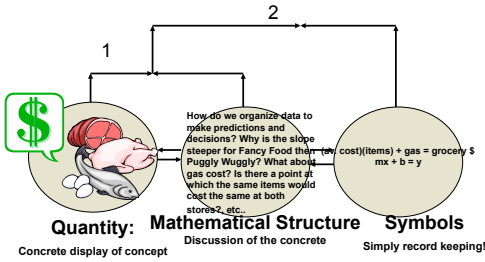
Even Algebra can fit this mold!



Faulkner adapting Leinwald, Griffin

Common Core Standards: Algebra

- Seeing **Structures** in Expressions
- **Creating** Equations
- **Reasoning** with Equations

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Algebra through Shopping

- The following lesson units/materials are all copyrighted with Wake County Public Schools/Valerie Faulkner.
- These have also been incorporated into Walch Educational publications, Intro to High School Math and Foundations of Mathematics
- Further training in these units is available through DPI.
- The reality that shopping can be modeled with algebra is, of course, public domain.
- Steve Leinwand also presents ideas using average cost from shopping register tickets and speeding ticket formulas to develop algebraic concepts.

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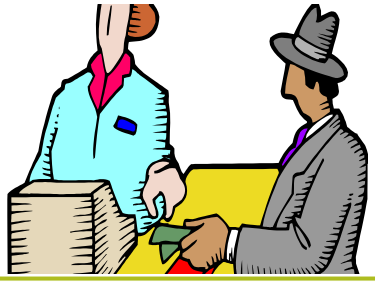
Unit 1
Concept Development Dollar Deals



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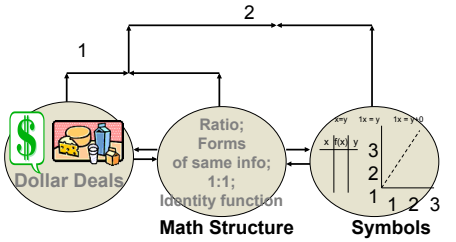
Dollar Deals Learning Objectives



Quantity: Concrete display of concept

Math Structure Discussion of the concrete

Symbols Simply record keeping!

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Dollar Deals

No Tax, No Tricks: One Dollar per One Item.

Where EVERYTHING is just one dollar.

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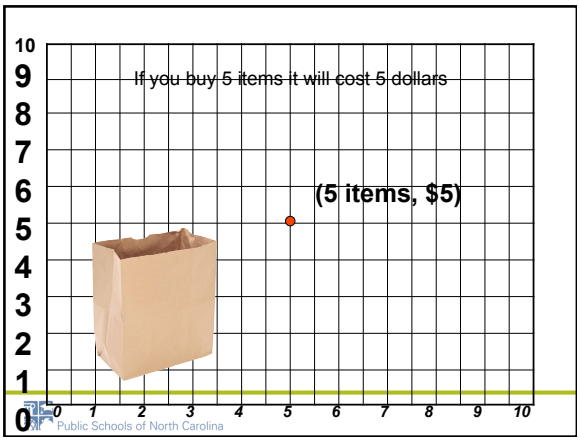

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X = Y

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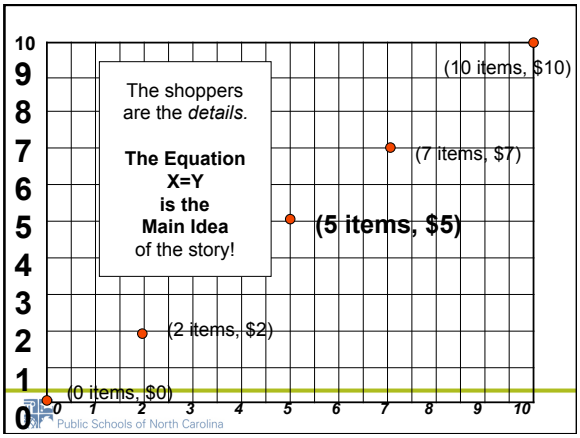

Unit 2
Dollar Deals vs. Puggly Wuggly

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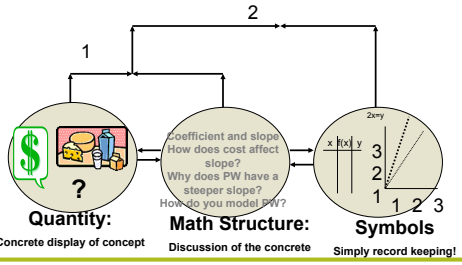
Puggly Wuggly vs. Dollar Deals Learning Objectives



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Puggly Wuggly Super Sale!

Bag day sale!

All items you can fit in 1 bag:
2 dollars per item!

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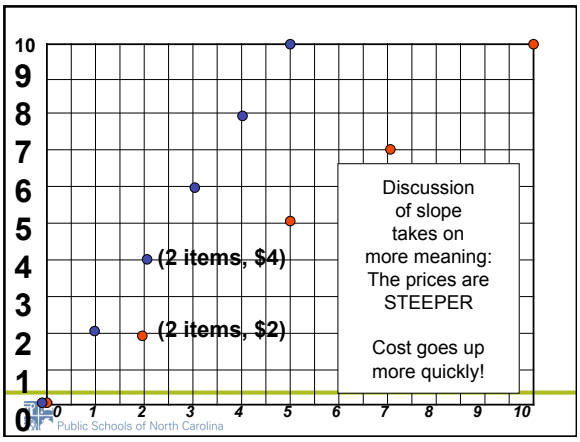

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Unit 3
Puggly Wuggly vs. Fancy Food

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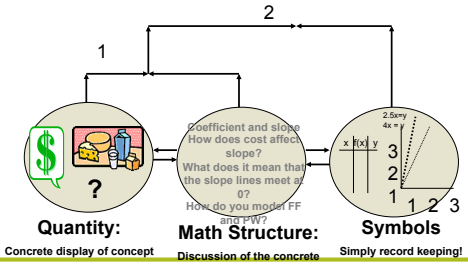
Puggly Wuggly vs. Fancy Foods Learning Objectives



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Puggly Wuggly

No sale—just our usual price:
$2.50 per item

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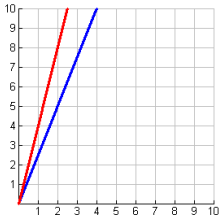
Fancy Foods

You can pay now, or you can pay later…
The best food at the worst prices!
All organic food
$4 per item

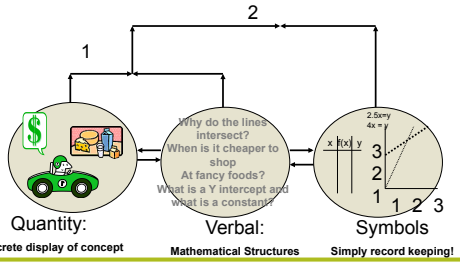
Other Forms of the Number Relationship!

X (Items)	Y=2.5x PW	Y=4x FF
1	2.5	4
2	5	8
3	7.5	12
4	10	16
5	12.5	20



Unit 4
Feeding the DumDee

Feeding the DumDee
Learning Objectives



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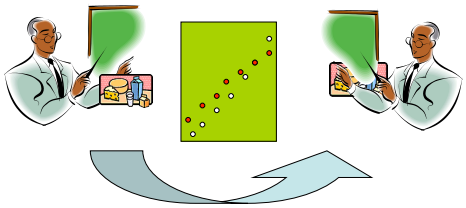
Feeding the DumDee
What Does it Look Like?



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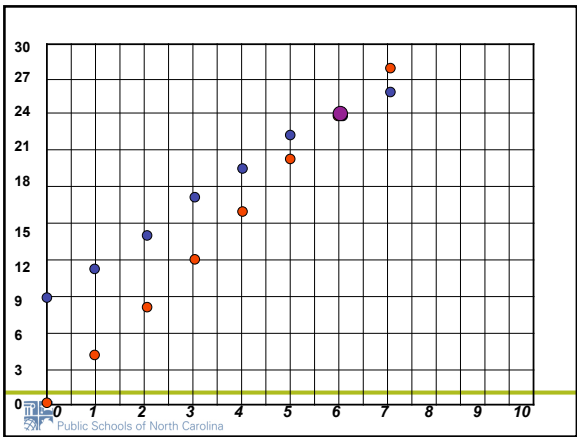

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Feeding the DumDee—Big Ideas

Students will begin to see what y = mx + b means in its entirety. Up until now our b has been the constant "0" and so it has "dropped out" of the equation. But now we have a constant other than 0.

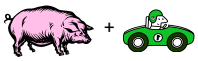
How does this affect our equation?

Our t-chart?

Our graph?

Our cost?

Mathematical Models
The Showdown:
PW + Gas vs. FF on foot

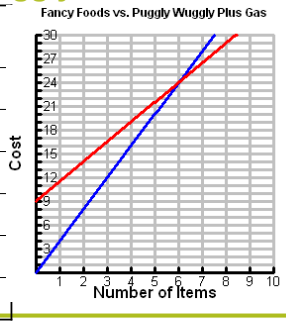


_____

_____

Fancy Food vs.
Puggly Wuggly + Gas!

X Items	Y=4x FF	Y=2.5x + 9 PW
1	$4.00	$11.50
2	$8.00	$14.00
3	$12.00	$16.50
4	$16.00	$19.00
5	$20.00	$21.50
6	$24.00	$24.00



Two shoppers, One purpose

Jenn and Jermaine had a little argument about shopping. Jenn said that she could shop for things cheaper than Jermaine, because "guys don't know how to shop." Jermaine thought that was ridiculous and challenged her to a shopping match. "You're on" she said. "Great" said Jermaine, "Save your receipt and you need to include the cost of travel to get to the store." Jenn agreed to these conditions and they were ready to go.

Jenn had an energy efficient hybrid car, so she felt confident that she would win, because Jermaine drove a truck. Puggly Wuggly was far away, but she thought the cheap prices at Puggly's $2/item bag day sale would be worth the drive. She spent $2.00 on gas for the whole trip.

Jermaine knew about the Puggly Bag Sale too. But he didn't drive his truck. He took the bus and it cost him $1.50.

Model the contest between the two with linear equations and then graph them. Answer the questions and be prepared to discuss with the class.

Mathematical Models

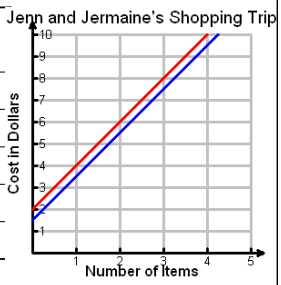
_____

_____

x Number of Items	Rule Math Model	y Cost	x Number of Items	Rule Math Model	y Cost
	+			+	

Jenn and Jermaine's Shopping Trip

X Items	Y=2x + 1.5 Jermaine	Y=2x + 2 Jenn
0	$1.50	$2.00
1	$3.50	$4.00
2	$5.50	$6.00
3	$7.50	$8.00
4	$9.50	$10.00



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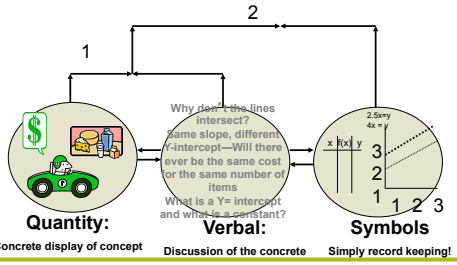
Feeding the DumDee--Jenn V. Jermaine Learning Objectives



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Given two points, find the equation of the line…

$$\frac{Y_1 - Y_2}{X_1 - X_2} \qquad \frac{\triangle Y}{\triangle X}$$

How are your students processing this?

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Why the Y?

$$\frac{\text{Cost}}{\text{Item}}$$

Cost ***per*** Item

$$\frac{\triangle \text{ Independent}}{\triangle \text{ Dependent}} \qquad \frac{\triangle \text{ Y}}{\triangle \text{ X}}$$

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How can I get from here:

(5,17)
(6, 20)

to here:

3x + 2 = y

with MEANING?

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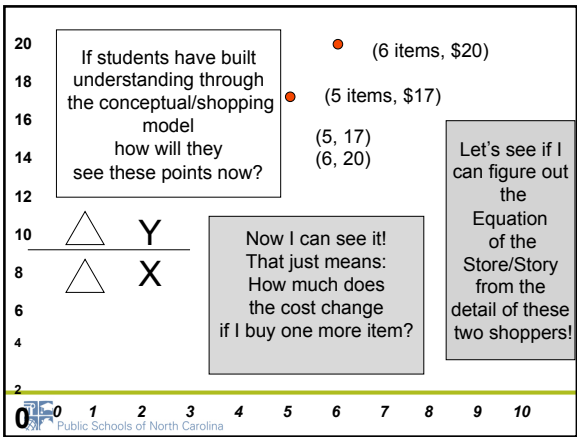

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- Seeing **Structures**
- **Creating** Equations
- **Reasoning** with Equations

Verbal:

You and your friends want to record a CD. The company you want to hire charges $300 for making the master and designing the art for the cover. It charges $2.50 for burning each CD. Use the function rule P(c)=300 + 2.5c. Make a table of values and a graph.

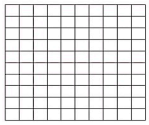
Algebraic

_____

Table

_____ (x)	Function: _____	_____ (y)

Graph



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$$Y = mx + b$$

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$$r = \frac{C}{2\pi} + \frac{100}{2\pi}$$

$$r = \frac{1}{2\pi} C + \frac{100}{2\pi}$$

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Y = mx +b

$$r = \frac{1}{2\pi} C + \frac{100}{2\pi}$$

Assignments from Units 7

Learning Task 7 –Painting Walls

Required Reading for Unit 9

(Be prepared to discuss in your groups.

"Mathematics Education in the 21st Century"

Deborah Ball

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Geometric Thinking

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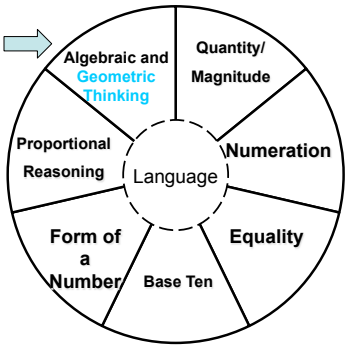

The Components of Number Sense

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Geometric Thinking

Defining the Concept

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Geometric Thinking Involves ---

- the study of geometric figures (shape and form) and their properties.
- congruence, symmetry, similarity and transformations.
- spatial measure.

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Diagnosis

Defining the Concept	**Diagnosis**	Where the Research Meets the Road	Classroom Application

Levels of Geometric Thinking

van Hiele Levels

- Level 0 Visual
- Level 1 Analysis
- Level 2 Informal Deduction
- Level 3 Deduction
- Level 4 Rigor

Van de Walle, John A. (2001) Geometric Thinking and Geometric Concepts

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Where the research meets the road

Defining the Concept	Diagnosis	**Where the Research Meets the Road**	Classroom Application

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Children enter school with a sense of informal knowledge of geometry.

"Instruction in Geometry needs to complement the study of number and operations in Grades Pre-K through 8."

Adding It Up ***National Research Council***

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Classroom Application:

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

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Foundation Level Training

Understanding the Relationship between Area and Perimeter

Using Griffin's Model

Mayer's Problem Solving Model

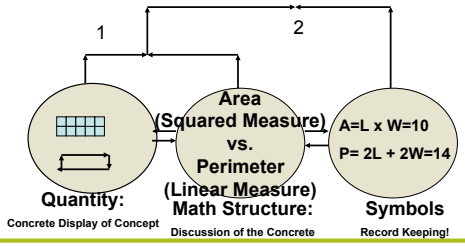
Components of Number Sense

Problem

MAX

- What are the whole number dimensions of a rectangle that can produce the greatest area given 150 feet of fencing?
- Context: There is a leash law at Atlantic Beach. What is the largest rectangular pen that could be built for Max with 150 feet of fencing for the times he is not on his leash?

Prototype for Lesson Construction



V. Faulkner and DPI Task Force adapted from Griffin

Cognitive Processes in Problem Solving Richard E. Mayer

- Translating
- Integrating
- Planning
- Executing

Cognitive Processes in Problem Solving
Richard E. Mayer

Translating

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Frayer Model for Linguistics

Whole Numbers

Definition
(in own words)
Counting Numbers plus 0

Characteristics
- **Rational**
- **Positive, except for 0**

Examples
0, 1, 2, 3, 4, …

Non-Examples
$\sqrt{2}$ $\frac{1}{2}$
-1, -2, ,-3 …;

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Frayer Model for Linguistics

Perimeter

Definition
(in own words)
The distance around an object (linear measure)

Characteristics
- Involves measuring in one dimension
- Involves computing with addition
- Involves problem solving

Examples
10 cm
2cm
Perimeter is 24 cm

Non-Examples
Area is 20 sq. cm².

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Frayer Model for Linguistics

Definition

(in own words)

The measure of the interior of a figure

(squared measure)

Characteristics

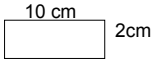
- Involves measuring in two dimensions
- Involves computing with multiplication
- Involves problem solving

Area

Examples

Area is 20 cm²

Non-Examples



Perimeter is 24 cm.

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Problem Solving-Translating

- I have 150 feet of fencing. When I use fencing, it will go around the distance of a figure (perimeter), in this case a rectangle.
- I know that rectangles have two pairs of opposite congruent sides.
- I know that when I am finding perimeter, I compute with addends.
- I know I can only use whole numbers and 0 cannot be the measure of a side.
- I know that there are many different pairs of sides that I can create with 150 feet of fencing.

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Problem Solving-Translating

- I know area is the squared measure of the interior of a figure.
- I know that the largest area is the figure whose interior measure is the greatest.
- I know that area can be represented by an array and I can use repeated addition or multiplication to solve for area.

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Cognitive Processes in Problem Solving
Richard E. Mayer

Integrating

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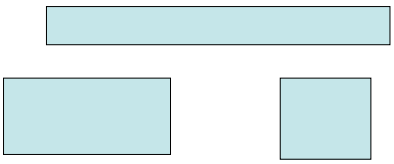
Problem Solving-Integrating

Building a mental model of the problem.



If the Perimeter is 150 feet,

which rectangle will have the greatest area?

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Cognitive Processes in Problem Solving
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Planning

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Let's Look at An Easier Problem

- Cafeteria Tables and Students
- Bag of 30 tiles

Cafeteria Tables

A one unit table will be able to seat four students.

A two unit table will be able to seat six students.

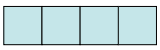
Following the pattern---

A three unit table will sit eight students.

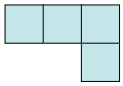
---even with this arrangement.

But what about---

A four unit table? It can seat ten students.



---even when arranged like this.



But what about this arrangement?

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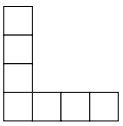
Challenge 1



Area = 7 **P =** 16

What other perimeter could be shown that has the same area?

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Challenge 2

A = 5 **P =** 12

Rearrange the area to show a diagram with a different perimeter.

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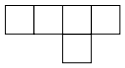
Challenge 3

Add a square so that the area is 6 and the perimeter remains 12.

A = 5 **P =** 12



Can you find other arrangements with an area of 6 and a perimeter of 12?

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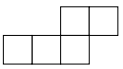
Challenge 4

Add squares so that the perimeter is 18.
What is the new area?

A = 5 **P =** 12



What other arrangements could be made to create different areas with the same perimeter?

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Looking at a Simpler Problem

Problem: What is the greatest rectangular area that can be produced with 20 feet of fencing?

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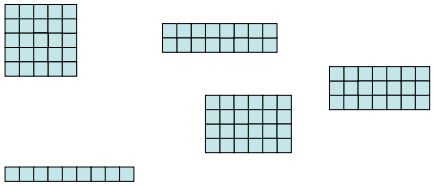
Problem Solving-Planning

Concrete- All Rectangles with a Perimeter of 20 with whole number dimensions.



Problem Solving-Planning

Representational/Verbal Discussion

- How can we organize the data?
- Which rectangle has the greatest area given the perimeter?
- What do you notice about the area of the rectangle as the sides increase or decrease?
- What rule can we find that will help us to find the greatest area of a rectangle with a given perimeter?
- How can we use the easier problem to find a pattern for the more difficult problem?

How Can We Organize the Data?

Length	Width	P= 2L + 2 W	A = L x W
1	9	P= 20 units	A = 9 sq. units
2	8	P= 20 units	A = 16 sq. units
3	7	P= 20 units	A = 21 sq. units
4	6	P= 20 units	A = 24 sq. units
5	5	P= 20 units	A = 25 sq. units
6	4	P= 20units	A = 24 sq. units
7	3	P= 20units	A = 21 sq. units
8	2	P= 20units	A = 16 sq. units
9	1	P= 20 units	A = 9 sq. units

Cognitive Processes in Problem Solving
Richard E. Mayer

Executing

Problem Solving-Executing

Symbolic

- Rectangle Dimensions
  - 1 x 9 = 9 square units
  - 2 x 8 = 16 square units
  - 3 x 7 = 21 square units
  - 4 x 6 = 24 square units
  - 5 x 5 = 25 square units (largest area!!!)

So what about---

150 feet of fencing?

Which Rectangle Will Yield the Greatest Area?

- A 1 foot by 74 foot rectangle?
- A 2 foot by 73 foot rectangle?
- ……etc.
- A 37 x 38 rectangle-----

Will Make Max the Happiest!

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Questioning through the Components of Number Sense

- Quantity & Magnitude
- Numeration
- Equality
- Base Ten
- Form of a Number
- Proportional Reasoning
- Algebraic and Geometric Thinking

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Foot Stuff -

Marilyn Burns

Exploring Area and Perimeter

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Area and Perimeter

- Draw a line around your bare foot on the graph paper.
- Figure the area of your foot and record the result inside the foot outline. (in square units)
- Place the string on foot outline and cut to same length (perimeter).
- Take your foot perimeter string and shape it into a square on the graph paper.
- Draw the outline (perimeter) of the square.
- Figure the area of that square and record the result inside the string square. (in square units)

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Area and Perimeter

- Are perimeters the same for both shapes?
- Are areas the same for both shapes?

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Clarifying the Conditions

- Three Cases
  - The length <u>or</u> width is increased
  - The length <u>and</u> width are increased
  - <u>Either</u> the length <u>or</u> width <u>increases</u> while the other dimension <u>decreases</u>

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Foot Stuff Questions

- What method did you use to figure the area of your foot?
- How did you figure out how to arrange your perimeter string into the shape of a square?
- How does the area of your foot compare to the area of the string square?
- Why do you think that this makes sense that the area of your foot and string square are different?

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Area & Perimeter

- How well do we/our students visualize a distance measurement vs. a squared measurement?
- How do we develop these concepts for our students?
- How do we prepare students for volume (cubed measurements)?

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Unit 8

REQUIRED READING

HANDOUTS

Assisting Students Struggling with Mathematics: RtI for Elementary and Middle Schools

IES Recommendations
Practice

Recommendation 1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.

Recommendation 2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.

Recommendation 3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

Recommendation 4. Interventions should include instruction on solving word problems that is based on common underlying structures.

Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.

Recommendation 8. Include motivational strategies in tier 2 and tier 3 interventions.

Assessment Suggestions for CC-K Classes

Sharon Griffin

May 28, 2006

Purpose of Assessment: To document the effects of NW in these classes.

Assessment Battery: I'm suggesting several measures, described below; that I think will be useful. All of them should be orally and individually administered to this age group and you will have to decide how many you want to include. Obviously, the more data you collect on children's pre-, post- and ongoing performance, the better you will be able to document the effects of the NW curriculum.

Adapting Tests for Non-Verbal Children: Many of the items for all tests at the level appropriate for this age group can be answered by pointing and, therefore, don't require a verbal response. The items that do require a verbal response often ask children to say a number (e.g. to tell you how many there are). If children are able to show you a finger display to represent how many, that would be an acceptable alternative. Alternatively, if children can point to a numeral on a number line to represent how many, that would also be acceptable alternative, as would any other way you can think of to allow non-verbal children to demonstrate their understanding. Whenever a response other than verbalizing the answer is used (e.g. finger display; pointing to a numeral) it should be recorded. Over time, children may acquire verbal competencies they didn't demonstrate at the outset and it would be valuable to have a record of these changes for each test item.

Number Knowledge Test: This test is a must. Many children in CC-K classes won't go beyond Level 0 of this test on the pre-test and it will take less than 5 minutes to administer to each child. This test is available in the NW Level A Assessment Book. I would administer this test to each child at the beginning of the school year (when children have become comfortable with school routines) and at the end of the school year. Because this test is closely linked to the learning objectives of the NW program and because

normative data is available, the results can be used to inform instruction and to measure each child's growth over the school year.

Time Knowledge Test: This test and the following (the Money Test) are optional tests. I would give them if you can manage it because they provide nice measures to assess transfer of learning and they take only a few minutes each to administer. The test items for the relevant levels of each test are included in the attached files. Like the Number Knowledge Test, these tests should be used as pre- and post-tests and should be administered at the beginning and at the end of the school year. Developmental norms are available for both of these tests. If you want to use them, I will send you an article or two to give you more information on them.

Money Knowledge Test: See above description.

NW Level A Placement Test: This test is included in the NW Level A Assessment Book. I would use it also as a pre- and post-test and administer it to each child at the beginning and at the end of the school year. If children do well on Level A at the beginning (unlikely) or the end of the school year, I would follow the instructions on the test and administer the Level B Placement Test.

NW Weekly and Cumulative Review Tests: These tests can be found in the NW Level A Assessment Book. They are designed to be formative assessment tools and to help the teacher determine whether a child should repeat some lessons from the week or the six cumulative weeks or whether the child has mastered the material sufficiently to move on. I would ask the teacher to keep these test sheets each time she uses them. At the end of the year, you can analyze them and document a number of things for each child (e.g. number of times a child had to take any one test before improving sufficiently to move on; progress on the test during the second or third administration). You can also relate these on-going assessments to the child's pre-test scores on the above measures to determine whether children who do well on the pre-test also progress smoothly throughout the program and master the material with relative ease. This may or may not be the

case and it may depend to a certain extent on other variables (e.g. teacher effectiveness; the other students in the class). This data would thus allow you to examine relative teacher effectiveness for a range of different learning objectives as well as the effects of different student bodies in each class (e.g. if one class has a lot of acting out students, this could slow down progress for all children).

NW Daily Assessment Tests: These tests, found at the end of each lesson, are called informal assessments. Teachers can simply check Yes or NO next to the behaviors listed for the activities taught that day or, better still, use the rubrics that are described on pages 9-11 of the Assessment Book. The rubrics allow the teacher to assign a score of 1 through 4 to each behavior and, thus, to compare student performance on the behaviors listed across time. This is another tool to document progress across the year. A student record sheet to record these scores for the whole year is included on page 100 of the Assessment Book.

E-Assessment Tool: I haven't seen this yet so I can't comment on it. It should be available before the start of the 2006-2007 school year.

Although this might sound like a lot of testing, the pre- post-tests can easily be administered in one session, lasting less than ½ hour, to most 5-year-olds. Ideally, they would be administered by a trained tester to ensure reliability across children and classrooms. If you are only assessing two classrooms, this should be a manageable task. The ongoing assessments should be administered by the teacher. Ideally, teachers would receive help (i.e. training) in administering these tests, in using them to guide their instruction, and in keeping good records of children's test scores. Once again, this is a manageable task.

POWERPOINTS

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Assessment / Interventions
Unit 8

- Overview of Assessment
- Framework for Assessment
- Models of Interventions

Is What Teachers Do equal to What teachers think?

- True! What we think will affect what we learn.
- Learning occurs instantaneously. T F
- You can only practice what you know. T F
- Proficiency builds gradually. T F

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"If the student has not learned – the teacher has not taught!"

- Missing or erroneous prior knowledge may be your biggest barrier
- Task difficulty is directly related to prior knowledge.
- It is not the task it is the skill!

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Let's Learn about Hacky Sack

- **Hacky Sack** is the trademarked name of a type of footbag. The name "hacky sack" came from the inventors of the footbag, John Stalberger and Mike Marshall. Marshall suffered a fatal heart attack in 1975, however Stalberger continued the business. At a later date, Stalberger sold the title to Wham-O.

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Moves

- **Inside Kick**
- **Outside Kick**
- **Toe Kick**
- **Knee Kick**

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RULES

- 1. **No Hands** (except when serving), **No Arms** - Shoulders are technically not allowed and are widely accepted among the average hack circle.
- 2. **Always serve the bag to someone else**, unless of course you are alone. Footbag is traditionally a game of courtesy, hence "The Courtesy Toss": a light lob usually toward the receivers knee.
- 3. **Don't bogart that bag.** - Don't always hog it 'till you drop it because that is bad for everybody else. Being able to pass well is important to almost all footbag games.
- 4. **Don't say "sorry".** Everyone makes mistakes, especially when learning, so sorries are unnecessary.
- 5. **Try not to give knee passes** Passes from the knee tend to go straight to the ground.

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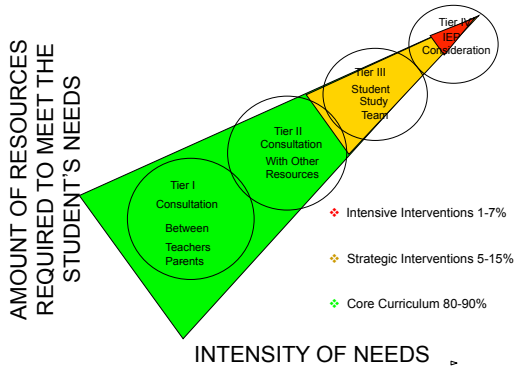
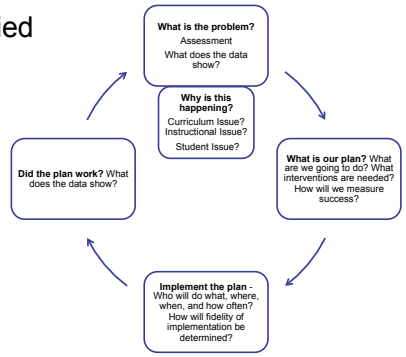
Learning Process

- Learning orientation vs. performance orientation (completing the task)
- How can you show which is important in your classroom?

How is all of this connected to the RTI Model?



Modified PSM



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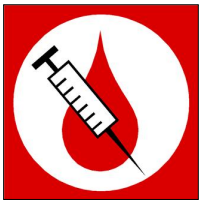
Fundamentals of Assessment



Universal Screenings

- Universal Screening is a general outcome measure used to identify underperforming students and to determine the rate of increase for students.
- Universal Screening (both high and low achievers may need interventions) G.O.M.s
- A Universal Screening will ***not*** identify why students are underperforming, rather it will identify which student is below / above the expected performance criteria for a given grade level in reading, writing, spelling, and math.

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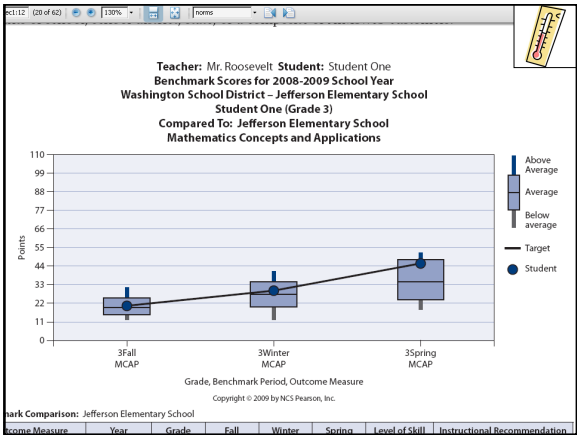

Number Knowledge Test

http://clarku.edu/numberworlds

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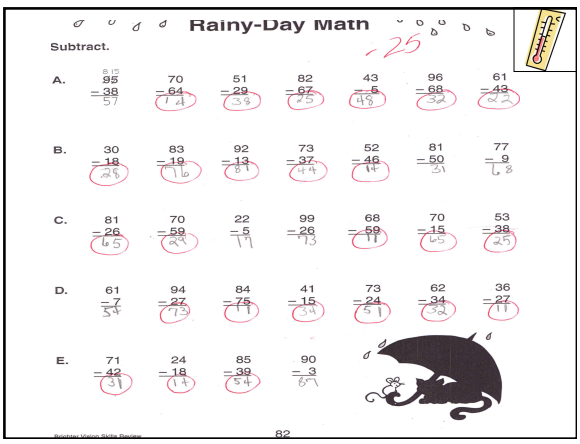


Rainy-Day Math

Subtract.

A. 95 − 38 = 57; 70 − 64 = 14; 51 − 29 = 38; 82 − 67 = 15; 43 − 5 = 48; 96 − 68 = 28; 61 − 43 = 22

B. 30 − 18 = 28; 83 − 19 = 16; 92 − 13 = 81; 73 − 37 = 44; 52 − 46 = 14; 81 − 50 = 31; 77 − 9 = 68

C. 81 − 26 = 65; 70 − 59 = 29; 22 − 5 = 17; 99 − 26 = 73; 68 − 59 = 11; 70 − 15 = 65; 53 − 38 = 25

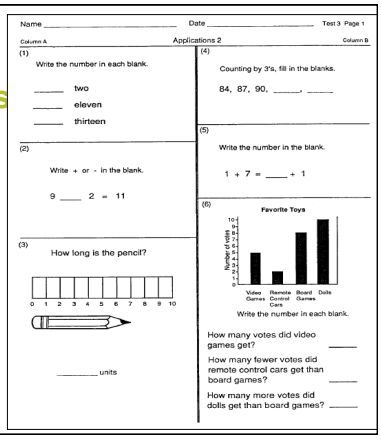
D. 61 − 7 = 54; 94 − 27 = 73; 84 − 75 = 11; 41 − 15 = 34; 73 − 24 = 51; 62 − 34 = 32; 36 − 27 = 11

E. 71 − 42 = 31; 24 − 18 = 14; 85 − 39 = 54; 90 − 3 = 87

Brighter Vision Skills Review

82

Concepts and Applications



Name _____ Date _____ Test 3 Page 1

Column A — Applications 2 — Column B

(1) Write the number in each blank.

_____ two
_____ eleven
_____ thirteen

(2) Write + or - in the blank.

9 ____ 2 = 11

(3) How long is the pencil?

0 1 2 3 4 5 6 7 8 9 10

_____ units

(4) Counting by 3's, fill in the blanks.

84, 87, 90, _____, _____

(5) Write the number in the blank.

1 + 7 = ____ + 1

(6) Favorite Toys

Number of votes: 0 1 2 3 4 5 6 7 8 9 10

Video Games, Remote Control Cars, Board Games, Dolls

Write the number in each blank.

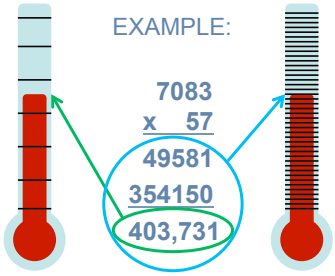
How many votes did video games get? _____

How many fewer votes did remote control cars get than board games? _____

How many more votes did dolls get than board games? _____

2nd Grade Example

Scoring – Sensitivity to Differences



EXAMPLE:

7083
x 57
49581
354150
403,731

By counting correct digits, it becomes easier to see small differences between students and to recognize incremental improvements in performance over time.

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Assessment for Diagnosis & Intervention Design (Tier II and III)

Data Sources:

- Student interviews
- Conceptual understanding
- Student work
- Skill-based measurements
- Mastery Measures

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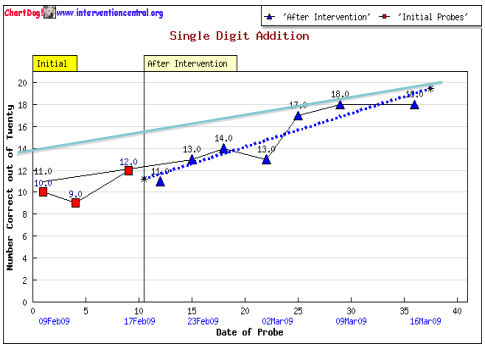

IES Recommendations Tier II and III

- Explicit and systematic
- Common underlying structures
- Visual representations of mathematical ideas
- Build fluent retrieval of basic arithmetic facts (at all grade levels for about 10 minutes)
- In-depth treatment of whole numbers (K-5) and on rational numbers (4-8)
- Progress monitoring
- Motivational strategies

Reviewing the Practice Guide

http://ies.ed.gov/ncee/wwc/pdf/practiceguides/rti_math_pg_042109.pdf

As a small group…

1. Review the IES Practice Guide
2. Consider ways in which the recommendations are related to the content of this training
3. Discuss how the practice guides might assist with intervention design and implementation

Intervention Ideas

- Subitizing
- eNumeracy Intervention Protocol using Number Worlds
- Number lines (games)
- Tens frames
- Structures of addition, subtraction, multiplication, division
- Diagram literacy
- Bar Models
- Conceptual story structures
- Sharon Griffin's Model of Instruction / CRA
- Hundreds boards (percentage, proportional reasoning , base ten)
- Hands on Equations

Intervention Ideas (con't)

How we TEACH:

- Trashketball
- Dollar Deals
- Paper Clip Chain
- Bean Party

Group work: Intervention Discussion

1. Read the intervention lesson
2. Discuss with the group:
   - What are your thoughts on the intervention?
   - What research backs this intervention?
   - For whom would this be applicable?

Web Resources for Assessment and Intervention

- Yearly Progress Pro
- http://www.mhdigitallearning.com/
- AIMS Web
- http://www.aimsweb.com/
- Intervention Central
- http://www.interventioncentral.org/
- Number Worlds Home
- http://clarku.edu/numberworlds/
- John Woodward, Transitional Math
- http://www2.ups.edu/faculty/woodward/home.htm
- Progress Monitoring
- http://www.progressmonitoring.net/
- National Center on Student Progress Monitoring
- http://www.studentprogress.org/chart/chart.asp

On-line resources to help with data

- http://www.thatquiz.org/
- http://www.aimsweb.com/measures-2/
- http://www.easycbm.com/
- http://enumeracy.com/

Tools to enhance interventions

- http://nlvm.usu.edu/en/nav/vlibrary.html
- http://illuminations.nctm.org/
- http://ies.ed.gov/ncee/wwc/
- http://ies.ed.gov/ncee/wwc/pdf/practiceguides/rti_math_pg_042109.pdf
- http://www.ncpublicschools.org/acre/standards/common-core-tools/

"Assisting Students Struggling with Mathematics": Response to Intervention (RtI) for Elementary and Middle Schools" IES National Center for Education Evaluation and Regional Assistance, NCEE 2009-4060, U.S. Department of Education

Ball, Deborah (1992) "Magical Hopes: Manipulatives and the Reform of Math Education", American Educator, Summer 1992

Ball's Website: http://wwwpersonal.umich.edu/~dball/

Fuchs, Lynn "The Prevention and Identification of Math Disability Using RTI", September 18, 2008 Presentation

Gersten, Russell, Clark, B, Jordan, N, Center on Instruction, "Screening for Mathematics Difficulties in K-3 Students" 2007.

Gersten, Russell, Jordan, N, Flojo, J., "Early Identification and Interventions for Students with Mathematical Difficulties", Journal of Learning Disabilities, Volume 38, Number 4, July August 2005

Gickling, Edward, PhD, Instructional Assessment in Mathematics, March 2003, Presentation at Exceptional Children's Conference

Griffin, Sharon. (2003). Mathematical Cognition, Royer, ed. Greenwich, CT.: Infoage Publishing.

Ma, Liping (1999) Knowing and Teaching Elementary Mathematics. Edison, NJ, Lawrence Erlbaum Associates.

Mayer, Richard (2003). Mathematical Cognition, Royer, Ed.. Greenwich, CT.: Infoage Publishing.

Unit 9

REQUIRED READING

HANDOUTS

Rubric for Lesson Plans

Focus	4	3	2	1	Total	
					L1	L2
Components of Number Sense	Lesson focuses on between 5 and 7 of the Components of Number Sense areas. The links to the Components of Number Sense are clear and strong.	Lesson focuses on between 3 and 4 of the Components of Number Sense areas. The links to the Components of Number Sense are clear and strong.	Lesson touches on 1-2 of the Components of Number Sense areas. The links to the Components of Number Sense are hard to identify and unclear.	Lesson does not address the Components of Number Sense.		
Language, Cognition, and Mathematical Structure	Multiple opportunities for two-way and in-depth discussion about the math (not just the procedures) is evident throughout the lesson	Opportunities for two-way discussion about the math (not just the procedures) are evident throughout the lesson.	Few opportunities for discussion about the math are given thoughout the lesson.	No opportunities for discussion about the math are given thought the lesson.		
Connects to Prior Knowledge	Multiple examples to foster in-depth understanding of the math are clearly evident throughout the lesson. The lesson ties into basic concepts and builds on prior knowledge.	Examples to foster understanding of the math are evident within the lesson. The lesson ties into basic concepts and builds on prior knowledge.	Examples to foster understanding of the math are somewhat evident within the lesson. The lesson may fail to tie into basic concepts and build on prior knowledge.	Examples to foster understanding of the math are not evident within the lesson. The lesson fails to tie into basic concepts and build on prior knowledge.		
				Grand Total		

Foundation Level Training Evaluation

On a Scale from 1 (strongly disagree) to 5 (strongly agree), please respond to the following.

1 2 3 4 5 The information in the training was valuable.

1 2 3 4 5 The pace of the training was appropriate.

1 2 3 4 5 The length of the training was appropriate.

1 2 3 4 5 The content of this training was appropriately challenging.

1 2 3 4 5 The presenters provided clear, useful and thorough information.

1 2 3 4 5 The activities, lectures, readings and assignments were well-balanced.

1 2 3 4 5 I am leaving the Foundation Level Training excited about my classroom instruction.

Comments:

POWERPOINTS

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Unit 9

Reflection

"Mathematics in the 21st Century: What Mathematical Knowledge is Needed for Teaching Mathematics"

Deborah Loewenberg Ball

1. What has the United States typically done to improve student learning and what should they be doing?
2. Why is increasing the amount of coursework in mathematics not enough?
3. How can the United States produce more effective mathematics teachers?

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Focus	4	3	2	1	Total	
Components of Number Sense	Lesson focuses on between 5 and 7 of the Components of Number Sense areas. The links to the Components of Number Sense are clear and strong.	Lesson focuses on between 3 and 4 of the Components of Number Sense areas. The link to the Components of Number Sense are clear and strong.	Lesson touches on 1-2 of the Components of Number Sense areas. The links to the Components of Number Sense are hard to identify and unclear.	Lesson does not address the Components of Number Sense.		
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				Grand Total		

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Looking Back

- Revisit your lesson plan.
- Choose one initial lesson plan per table to discuss as a group.
- Using your rubric in your handout section, assess the initial lesson plan with a rubric score. (1st Column)
- Discuss as a group how you can make the initial lesson plan stronger as it relates to the content of this training.

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Looking Forward-Lesson Revision

- **Look at the language and mathematical structure in your lesson.**
- **Is there a better way to talk about the connections you are trying to make?**
- **How can you make this lesson stronger with regard to connecting to prior knowledge and addressing previously learned skills.**
- **Can we use multiple strategies?**
- **Remember the Sharon Griffin model. Can we move the student through all areas of the prototype for lesson construction?**
- **Think about the Components of Number Sense.**
- **What questions can you ask your students from as many of the components as possible to connect students to the math that they already know?**

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Lesson Revision (See LT 9 Rubric.)

- **As a group, list at least five changes you could make to the lesson to strengthen the instruction for the student.**
- **Write a brief reflection as it relates to the strengths of the new lesson.**
- **Calculate a rubric score for the revised lesson.**

Submit: **(Be sure to include names of the participants in your group.)**

- **Lesson Plan with revisions**
- **Rubric with scores for both lessons**
- **Reflection**

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