CBMS

Nat'l Summit on the Mathematical Education Of Teachers


Opening Session - "Challenges in the Mathematical Education of Teachers: Why is the Preparation of Mathematics Teachers so Difficult?"

Remarks by Liping Ma
Arithmetic in American Mathematics Education: An Abandoned Arena?

In American elementary mathematics education, *arithmetic* is viewed as negligible, sometimes even with pity and disdain—like Cinderella in her stepmother's house. Many people seem to believe that arithmetic is only composed of a multitude of "math facts" and a handful of algorithms. If some voices want to argue that there are more than math facts and algorithms in arithmetic, they may mention that it also contains the meanings of the four basic operations—addition means putting together, subtraction means taking away, multiplication is repeated addition and division is repeated subtraction. Who would expect that the intellectual demand for learning such a subject actually is challenging and exciting? People seeking more "intellectually-demanding" mathematics for our young students look beyond arithmetic to probability and geometry.

From my perspective, however, there are significant potentials of arithmetic in mathematics education that are overlooked in this country. I would like to share with you some of my observations.

During the last few years I have been asking my American acquaintances in education, among them quite a few mathematics teachers and teacher educators, a simple question: "What do you call something like this: '3 + 2 = ' in English?" I stumbled upon this question by accident. One day when I wanted to discuss teaching and learning of word problems with people, I noticed that there was an important term missing in my English vocabulary. In China, to solve a

word problem it takes a process of three steps. Take, for example, the following word problem:

> John made 3 paper airplanes and Mike made 2 paper airplanes.
> How many airplanes did they make in all?

The first step in solving the problem, which is considered the key step, is called "**lie shi**". In this step one investigates the problem carefully and then writes out "3+2=", or, "2+3=". The second step is "computing"—to compute and get the result, which, in this case, is to write the number "5" after the equal sign. The third step is "answering"—to write an answer with the following format: "Answer: They made 5 airplanes in all."

For Chinese teachers and students, "lie shi" is the key part to solving word problems. If students get the "lie shi" correct but the computation wrong, their work is considered partially correct and partial credit may be given. However, if they get "lie shi" wrong but somehow the answer "correct", their work is considered totally wrong and no credit will be given. The word problems in Chinese math textbooks tend to have relatively simple numbers so that students' main effort will be focused on "lie shi" rather than computation.

I was struggling to find a translation for the term "lie shi". Translating the first half of the term, "lie," was easy—I knew that as a verb it means to form, to make, or to compose, etc. Yet I was stuck with the second half, "shi": How does one describe "3+2=" in English? I had no idea how to express this most common "thing" we see in elementary mathematics. I searched in my collection of current mathematics textbooks, but could not find a proper term. Then I started to ask around. Some people told me that they did not know what to call it either. Others gave me their answers. The answers, however, were different. The following are the five conversations prompted by my question (the letter "M" stands for me and the letter "R" stands for "reply"):

<u>First answer: "math problem"</u>

R: Umm…This is a math problem.

M: Okay… But what kind of math problem? Some math problems we call "word problems", such as this (I make up a simple word problem). But what do you call a problem like "3 + 2 ="?

R: Umm…You know what? I really have no idea.

<u>Second answer: "math sentence"</u>

R: Umm …I would say that it is a "math sentence".

M: So, for doing the computation of "3 + 2 =" you say "to compute a math sentence"?

R: Umm …It seems that “computing a sentence” doesn’t make much sense. Interesting…I never had thought about this.

Third answer: “number sentence”

R: Umm …I would say that it is a “number sentence”.

M: So, for doing the computation of “3 + 2 =” you say “to compute a number sentence”?

R: Umm …I am not sure.

Fourth answer: “horizontal problem”

R: It is a horizontal problem.

M: Horizontal problem?

R: There are vertical problems like what we do with columns. And this is a horizontal problem.

Fifth answer: “equation”

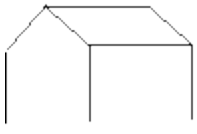
R: I would call it an equation.

M: But an equation should have numbers or variables on both sides of the equal sign, right?

R: Umm …Then I don’t know what is its correct name.

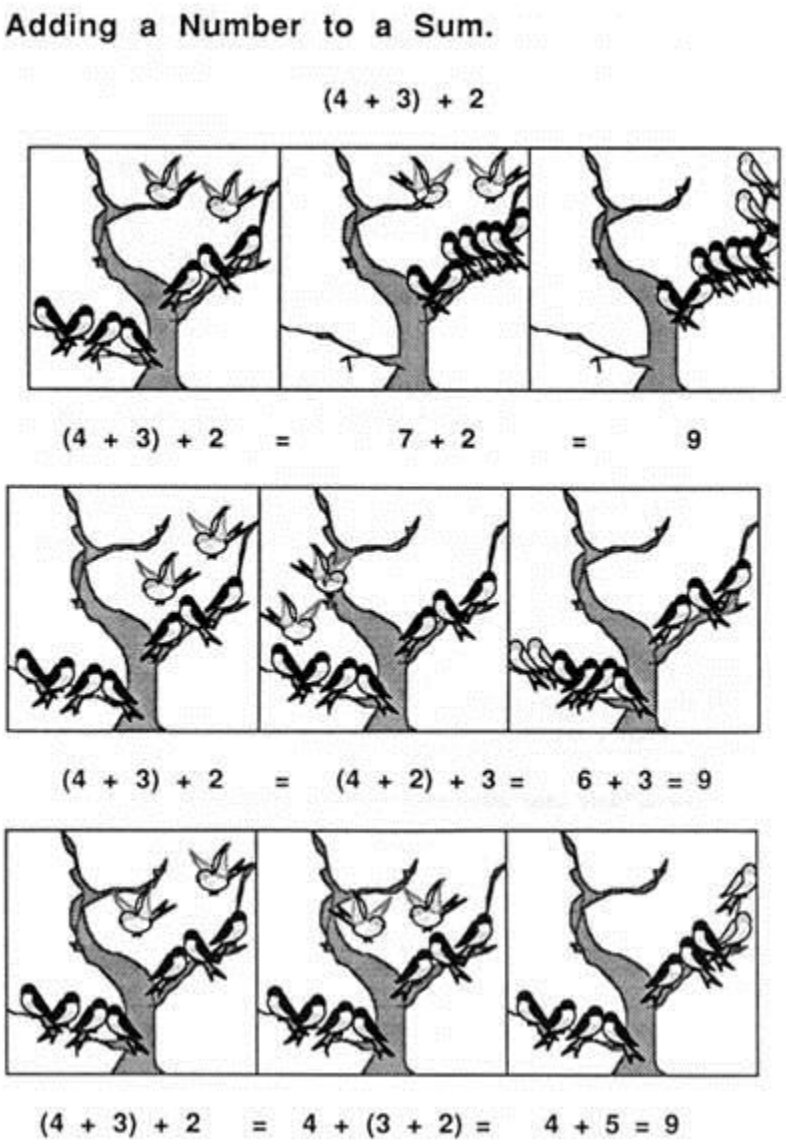
The lack of consistency and confidence in the replies led me to the conclusion -- this term, “shi” is actually not receiving attention in current American mathematics education.. Last fall, I finally found something convincing. In an old American textbook called “*Sheldon’s Complete Arithmetic*” published in 1886, the “thing” like “3 + 2 ” was called a “**mathematical expression**”, or “**expression**”. The process of computing the “thing” like “3 + 2” is called “**to find the value of an expression**”. After a few years of searching, now I know how to translate the term “lie shi”—it should be “to compose a mathematical expression.”

“To compose a mathematical expression” has potential significant pedagogical power in mathematics teaching and learning. It is widely used in math education in other countries. Let’s see some examples. In a first grade math class I visited in China, the teacher was teaching the number “8”. She asked each student to make a figure with 8 sticks in their kit of manipulatives. Then she asked them to compose math expressions with the sum 8 out of the figure they made. For example, out of the following house made by a student, the class might compose expressions like 4+4= , or, 5+3=, etc. Then she asked students to explain the expressions they composed. Students replied that if you take the



left side of the house as one part and the rest as another part, you get 4+4=; if you take the roof as one part and the rest as another part, then you get 5+3=, etc. By composing mathematical expressions out of the figures, the young students are learning how to model the number relationships.

The following is a page from the first grade Russian math textbook (translated by the University of Chicago School Mathematics Project). From it we can see the important role that math expressions are playing in mathematics education:



The following is a word problem from a 3rd grade Russian math textbook:

- 2 televisions and 4 radios were purchased for a resort building. 756 rubles were paid for everything. The price of a television is 270 rubles. How much does a radio cost?

Mathematical expression: (756 − 270 × 2) ÷ 4 =

The following two word problems are from "*Sheldon's Complete Arithmetic*":

- How many bushels of rye at 88 cents per bushel must be given for 33 hogsheads of molasses, each containing 63 gallons, at 80 cents per gallon? (p. 108 #27)
  Mathematical expression: (80 × 63 × 33) ÷ 88 =

- With $^{3}/_{4}$ of what a man received for a ship he paid for $318^{3}/_{4}$ acres of land at $180 per acre, and for buildings costing $13,125; what amount did he receive for the ship? (p. 108 #25)
  Mathematical expression: (180 × $318^{3}/_{4}$ + 13,125) ÷ $^{3}/_{4}$=

The following is a word problem from a current 5th grade Singapore math textbook:

- Mrs. Chen made some tarts. She sold $^{3}/_{5}$ of them in the morning and $^{1}/_{4}$ of the remainder in the afternoon. If she sold 200 more tarts in the morning than in the afternoon, how many tarts did she make?
  Mathematical expression: 200 ÷ ($^{3}/_{5}$ – (1– $^{3}/_{5}$) × $^{1}/_{4}$) =

Through the above examples I would like to make two points:

1. Arithmetic is intellectually challenging and pedagogically powerful for elementary students;
2. The "math expression" is only one of quite a few important concepts I have noticed missing in current math education in this country.

How was the arena of arithmetic, as taught in other countries, abandoned in the U.S., and why? I believe that there must be some positive reasons that made it happen. However, serious reflections on this issue need to be conducted.