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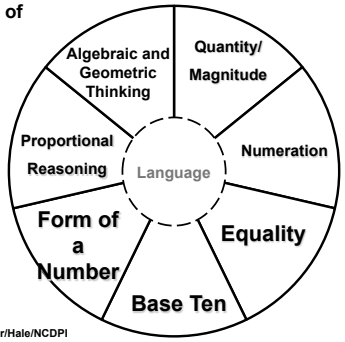
Unit 6
Base Ten, Equality and Form of a Number

IDEAs that Work
Office of Special Education Programs

8.8.13

The Components of Number Sense



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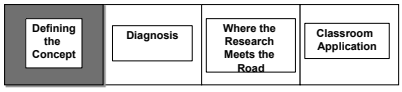
Equality

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Equality is a mathematical statement of equivalence of two quantities and nothing more.

Cain, Faulkner, Hale 2007

Base Ten



Base Ten-Defined

- Base ten digits are often used with a decimal separator and includes the start of a fractional point and positive and negative numbers.
- Linguistically, when we use English, the structure of the number words shows a base of ten, at least at the outset. When we write numbers, the structure of our number symbols also shows base ten.
- Sign language mimics our language not our number system.

Walter S. Sizer Base and Subbase in a Number System

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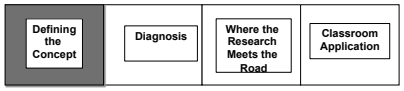
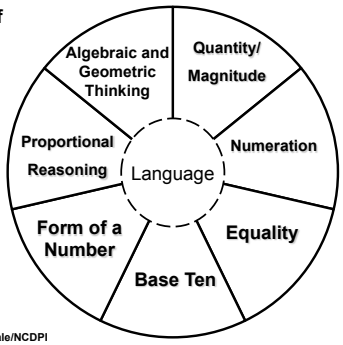
Form of a Number



Form of a number

Form of the number can be defined as multiple representations of quantity, ratios, and mathematical information.

The Components of Number Sense



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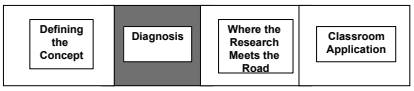
Diagnosis



Base Ten

- Has the student been exposed to base ten concepts?
- What are the early signs of base ten issues?
- Math Student Profile Checklist

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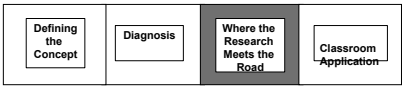
Where Research Meets the Road



Sharon Griffin's developmental nature of children using two number lines.

- 6 year old structure
- 8 year old structure
- 10 year old structure

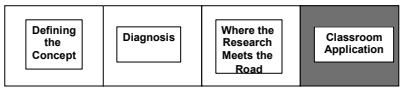
Piaget's developmental nature of learning

- Motor (0-2)
- Preoperational (2-7)
- Concrete (7-11)
- Formal (11-Adult)

Vygotsky - Zone of proximal development

- Constructivist
- We can move students through Piaget's stages more quickly depending on the types of activities in which engage them.
- Learning happens just above the mastery level. (ZPD)

Classroom Application:



Base Ten

- Sharon Griffin
- Liping Ma
- John Woodward

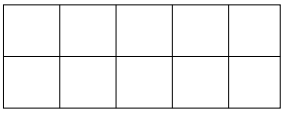
Math Facts

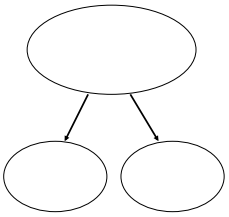
8 + 5
Fact Families?
Flash Cards?
Rewards?

Problem?
We see this as a one-step building block.

Cognitively, it is not.

Making 10: Facts within 20



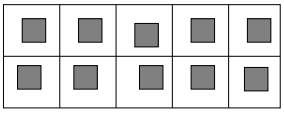


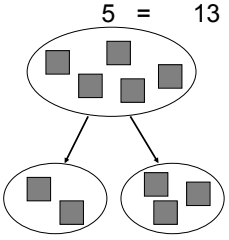
Makes 10 Left over

_____ + _____ = _____

Making 10: Facts within 20

8 + 5 = 13





Associative Property of Addition

__8__ + __(2 + 3)__ =

Makes 10 Left over

(8 + 2) + 3=

(10) + 3= 13

Base Ten

My student can't compute 42 plus 10. How do you remediate this student?

My student subtracts 108 – 19 and arrives at an answer of 89, but cannot explain their answer. Does that tell you that they understand base ten? How would you know?

Base Ten

- My student is multiplying 20 x 3 and writing the entire problem out and solving it procedurally and getting it correct. How do you respond?
- My student divides 108 by 9 and arrives at 2 for the quotient? How do you respond?

Base Ten

- You have a student who struggles with memory and clearly cannot do 2-digit by 3-digit multiplication. She is a seventh grader and must be able to perform computations with decimals, fractions and percent.
- How do we help her to use base ten to allow her access these SCOS skills?

Transitional Math
John Woodward's Strategies

Building Number Sense

John Woodward Resources

Addition Number Sense
Base Ten

43 + 12 → 40 + 3 + 10 + 2

40 + 10 + 3 + 2

50 + 5

Answer: 55

53	50	3	
+ 15	10	5	
	60	8	→ 68

Addition with Regrouping

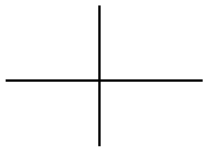
29	20	9	
+ 15	10	5	
		14	
	30	10 + 4	→ 44

10

	20	9	
	10	5	
	40	4	→ 44

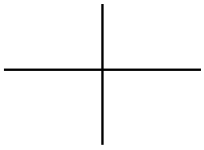
Try It!

49 + 37 →



Decompose both numbers into tens and ones

Combine ones



Trade group of ten ones for 1 ten.

Combine tens.

Answer →

Subtraction

76
- 29

70	6
- 20	9
60 + 10	6
- 20	9
60	6+10 (16)
- 20	9
40	7

Answer: 47
Estimate: 80 – 30
Calculator Check: 47

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You Try It!

81
- 52

Estimate:
Calculator:
Answer:

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95 → 90 | 5

x 3 x | 3

15

270

285

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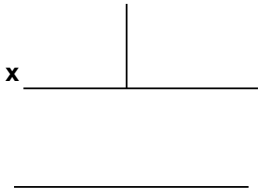
Try It!

83 x 7



What about Larger Numbers?

- **Sense making of the numbers.**
- **In the real world we use a calculator to at least check our work.**

Use Estimation to get a ballpark number.
389 x 78 ~ 400 x 80 =

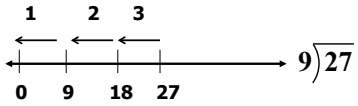
4 x 100 x 8 x 10=
4 x 8 x 100 x 10= 32 x 1000=32,000
297 x 31 ~ 300 x 30 = 9000

Division

- Use a Number Line
- Use Extended Facts
- Use Estimation

Division (Works with facts and Conceptually Guided Operations.)

Use of the Number line



$9\overline{)27}$

Division

- Estimation
  - Promotes number sense
  - Gives students an explicit strategy to check solutions to problems they work on a calculator.
  - Uses the strategy of near fact.
  - Helps with two digit multiplication by removing power of tens.

$3\overline{)48}$

$5\overline{)\overset{10}{50}}$

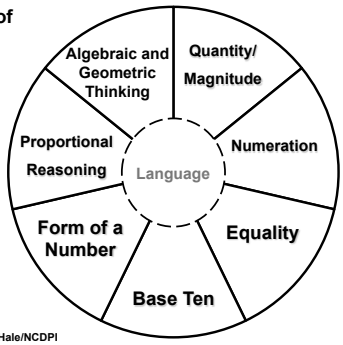
$34\overline{)239}$

$30\overline{)240}$

$3x10\overline{)24x10}$

$3\overline{)\overset{8}{24}}$

The Components of Number Sense



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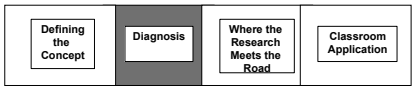
Diagnosis

Equality/Forms of a Number



Diagnosis Strategies

- What kinds of experiences have students had with understanding and exploring equality/forms of a number?
- Can the student tell you what equality is?
- Can they recognize different forms of a number?
- Does the student have an internal sense of a balance scale?
- Can students explain what it means that these representations are different forms of equal values?
  - Example .45 = 45/100 = 9/20=45%

Assessments
Number Knowledge Test
Informal probes (Craven County Probes)

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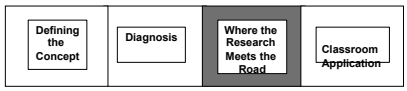
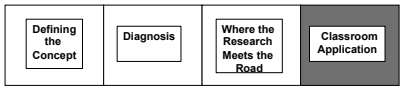
Where Research Meets the Road



Research Equality / Form of a Number

- What does the research say about using equality and form of a number
- TIMSS (1999, 2003)
- Ball, 2006
- Ma, 1999

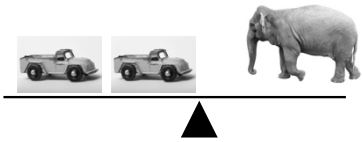
Classroom Application:



Are these equal?

$$1 = 1^3$$

Are these the same?



Are two cars equal to two cars?
Is one elephant equal to two cars?
Are these the same?

What about this?

X = Y on a balance scale

3x + Y = 0 on balance scale

What else could you put on the left hand side the balance?

- Discuss in small groups base ten concepts for addition, subtraction, multiplication, division, decimals and percent.
  - Assignments by tables of concept
  - Report out by group
  - Now try it!
    - Change the way we talk to kids
    - Role playing with student mistakes.

Addition

1) 16 + 7 = 113

2) 14 + 3 = 17

3) 35 + 81 = 116

4) 62 + 8 = 610

5) 407 + 63 = 4610

6) 569 + 724 = 12813

Subtraction

34 − 2 = 212

86 − 7 = 79

71 − 69 = 2

42 − 27 = 15

56 − 51 = 115

854 − 60 = 7814

Multiplication

1) 17 x 5 = 205

2) 40 x 8 = 320

3) 23 x 4 = 122

4) 27 x 31 = 27 + 121 = 1237

5) 54 x 19 = 726 + 54 = 1266

6) 56 x 28 = 728 + 122 = 1948

Division

1) $2\overline{)86}$ $\begin{array}{r}34\\ 8\\ \hline 6\\ 6\\ \hline \end{array}$

2) $4\overline{)20}$ $\begin{array}{r}5\\ 20\\ \hline \end{array}$

3) $7\overline{)133}$ $\begin{array}{r}91\\ 7\\ \hline 63\\ 63\\ \hline \end{array}$

4) $3\overline{)619}$ $\begin{array}{r}62\\ 6\\ \hline 19\\ 18\\ \hline 1\end{array}$

5) $8\overline{)816}$ $\begin{array}{r}201\\ 8\\ \hline 16\\ 16\\ \hline \end{array}$

6) $6\overline{)678}$ $\begin{array}{r}311\\ 6\\ \hline 7\\ 6\\ \hline 18\\ 18\\ \hline \end{array}$

Decimals

1) $\begin{array}{r}24.3\\ +\ .59\\ \hline 30.2\end{array}$ 2) $\begin{array}{r}6.7\\ +\ .88\\ \hline 15.5\end{array}$ 3) $\begin{array}{r}4.52\\ +\ .078\\ \hline 5.30\end{array}$

4) $\begin{array}{r}379.432\\ +\ 23.556\\ \hline 61.4992\end{array}$ 5) $\begin{array}{r}72.34\\ +\ .6672\\ \hline 1.3906\end{array}$ 6) $\begin{array}{r}8.216\\ +\ .797\\ \hline 16.186\end{array}$

It's All About the Story

- You say 13-5, and the student responds by placing 13 ones on the base ten mat and then takes away 5. How should the teacher respond?
- A student says that 0.5 + 0.3 and 0.4 + 0.4 are the same problem. How do you respond?
- A student says 3 – 2 is the same as 3 + -2. How do you show the student that these statements are not the same, but they have the same result?
- I asked the student if 0.50 and 50% is the same thing and they say no. How do you respond?

References

- Ball, 2006
- Ma, 1999
- John Woodward Transitional Math
- TIMSS (1999, 2003)

Assignments from Units 5 and 6

- **Learning Task 5 (See Rubric)**
  - **Structures of Addition, Subtraction, Multiplication and Division**
- **Learning Task 6 (See Rubric)**
  - **Number Knowledge Test with Reflection**