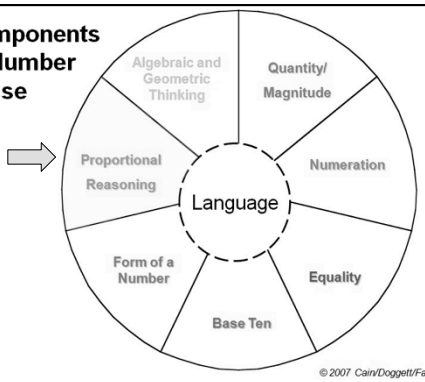
Public Schools of North Carolina

Unit 7
Proportional Reasoning
Algebraic and Geometric Thinking

IDEAs that Work
Office of Special Education Programs

8.8.13

Components of Number Sense



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Proportional Reasoning

Defining the Concept

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Proportional Reasoning

- Proportional reasoning involves a multiplicative relationship between two quantities.
- Proportional reasoning is one of the skills a child acquires when progressing from the stage of concrete operations to the stage of formal operations.

Language

FUNCTIONAL

The relationship of two values in the SAME measure space.
We utilize this internal multiplicative relationship to *maintain proportionality* and/or similarity.

e.g.: A picture 'blown-up' to a new size using the known relationship within the original picture size to figure out possible size options

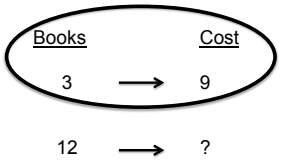
SCALAR

The relationship of values from two DIFFERENT measure spaces
We utilize this corresponding multiplicative relationship to move between different SCALES or to extrapolate from one situation to another

e.g. A picture being re-sized from one width to another width using the relationship between the original width and the desired width to create the newly-sized picture

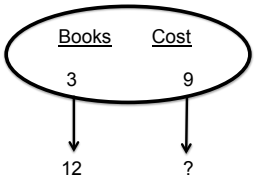
Measure Space

Functional – within one object or 'story' or measure space



The relationship of the values within the SAME measure space (situation).

Scalar – between two objects or 'stories' or measure spaces

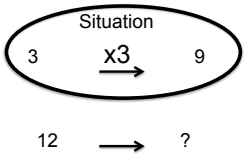


The relationship between two corresponding values in the DIFFERENT measure spaces (situations).

"Use of Schematic Representations in Improving Children's Understanding of Proportional Reasoning Problems" Pyday and Nunes, University of Oxford

Measure Space

Functional – within one object or 'story' or measure space



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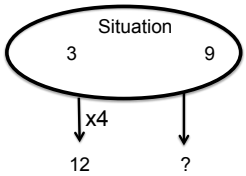


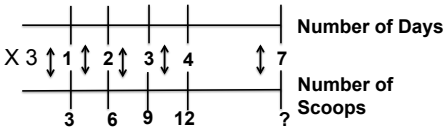
The relationship between two corresponding values in the DIFFERENT measure spaces (situations).

Adapted from "Use of Schematic Representations in Improving Children's Understanding of Proportional Reasoning Problems" Pyday and Nunes, University of Oxford

Functional

Use what is known within a situation to figure out missing values for an analogous situation…

A hamster eats 12 scoops of food in 4 days. How much food will the hamster need for 7 days?

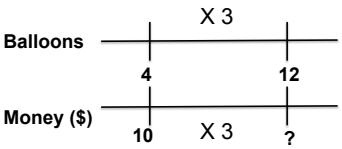


"Use of Schematic Representations in Improving Children's Understanding of Proportional Reasoning Problems" Pyday and Nunes, University of Oxford

Scalar

Use the relationship of known **corresponding parts of an analogous situation** to extrapolate and find the new space missing part…

Emily bought 4 balloons and paid $ 10 for them. She went back and bought 12 more balloons. How much did she have to pay for the 12 balloons?



"Use of Schematic Representations in Improving Children's Understanding of Proportional Reasoning Problems" Pyday and Nunes, University of Oxford

Functional

Use the relationship existing within the original measure space (situation)

3 feet long	**=**	**? feet long**
2 feet wide		**? feet wide**

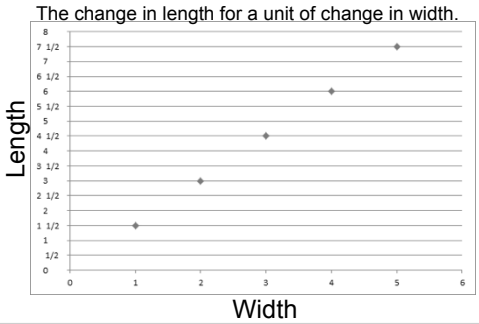
Here, the Length is 3/2 times the size of the width
The width is 2/3 times the size of the length

To create a correlate in a second space we consider the relationships from the same space:

To build a second space length from a second space width we multiply by 3/2
To build a second space width from a second space length we multiply by 2/3

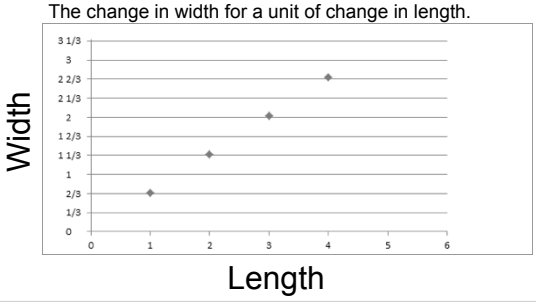
Rate of Change

The change in length for a unit of change in width.



Rate of Change

The change in width for a unit of change in length.



Scalar – Use the relationship that exists between two different measure spaces

A rectangle has a length of 3 feet and a width of 2 feet. How wide is a similar rectangle that is 3 feet longer in length than the original rectangle?

$$\frac{\text{3 feet long}}{\text{2 feet wide}}\left(\frac{2}{2}\right)=\frac{\text{6 feet long}}{\text{? feet wide}}$$

Our scalar operator is 2 because the relationship between the known analog parts is 2.

You Try. Use Functional and then Scalar to figure out Eli's cost

Grant and Eli each bought the same kind of bubble gum at the same store. Grant bought two pieces of gum for six cents. If Eli bought eight pieces of gum, how much did he pay?

	Bubble Gum (pieces)	Cost (cents)
Grant	2	6
Eli	8	?

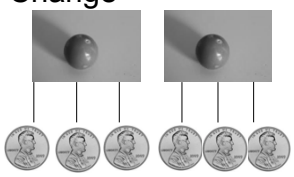
Functional Method: SLOPE

The relationship of the values within the SAME measure space (situation).

Constant Rate of Change

2G : 6P = 8G : 24P

2G : 8G = 6P : 24P



Proportional Reasoning: Student Misconceptions and Strategies for Teaching, PCK Tools, CAMS and CORE

Scalar Method: MULTIPLICATIVE IDENTITY

The relationship between two corresponding values in the DIFFERENT measure spaces.

Scalar operator ⟹ Form of ONE

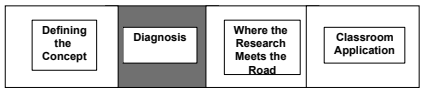
2G : 6P = 8G : 24P

2G : 8G = 6P : 24P

$$\frac{2G}{8G} \times \frac{3}{3} = \frac{6P}{24P}$$

Proportional Reasoning: Student Misconceptions and Strategies for Teaching, PCK Tools, CAMS and CORE

Diagnosis



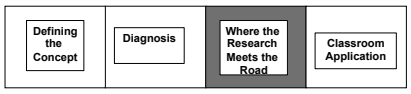
Proportional Reasoning---

- Is foundational to higher mathematics
- Does not always go hand-in-hand with symbolic representation
- Has four levels of solution strategies
  - 0 (use of additive strategies, solution arrived at by luck)
  - 1 (use of pictures, models, manipulatives)
  - 2 (use of level 1 strategies and multiplication and division strategies as well)
  - 3 (use of cross-multiplication or equal ratios)

Student Issues

- Students do not analyze the relationships among symbols.
- Students use the cross-product method even when the within or between relationships are obvious.

Where the research meets the road



"We view proportional reasoning as a pivotal concept. On the one hand, it is the capstone of children's elementary school arithmetic; on the other hand, it is the cornerstone of all that is to follow."
(Lesh, R., Post, T., & Behr, M. (1988)

Four Major Issues

- Emphasis on procedures vs. developing sense of the meaning of rational numbers
- We teach as if Level 3 were the access point, rather than Level 1
- Use of representations in situations where rational numbers and whole numbers are easily confused.
- The inherently confusing nature of fractional notation to represent proportions
  - (2 for every 5, 2 out of 5, and 2/5)

Joan Moss and Robbie Case 1999

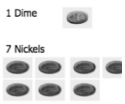
The inherently confusing nature of fractional notation to represent proportions
Joan Moss and Robbie Case 1999

1/6 or fraction of stable unit like money (9/20) (Story One)
VERSUS
1 out of 6 or proportion of a changing unit (8/30) (Story Two)

How Do We Represent the Money?

How do we determine this ratio?

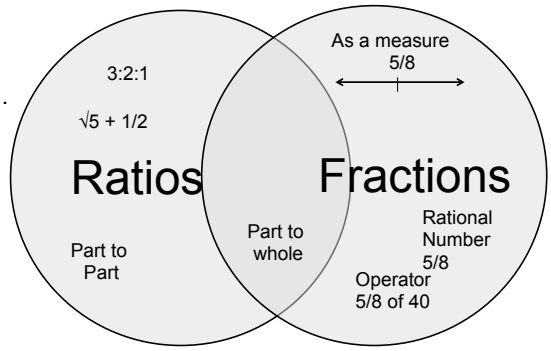
_45_ /_100__



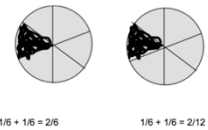
How Did the Team Do Altogether?

How do we determine this ratio?

__8_ /_30_



FRACTIONS MATTER:
Teaching Fractions <u>must include</u> connections to Proportional Reasoning!



What is my Unit?
I can't make sense of a fraction unless I know what my comparative unit is!

How does my **portion** size affect my servings?

Solutions

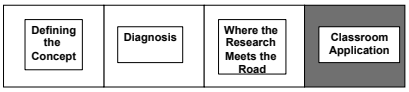
- Greater emphasis on meaning
- Greater emphasis on the proportional nature of fractions
- Greater emphasis on Level 1 and Level 2 solution strategies
- Greater development of diagram literacy

Joan Moss and Robbie Case 1999

Proportional Reasoning

- Develops hand in hand with Quantity and Magnitude.
- Is a skill the child obtains when moving from the concrete to the formal operational stage of development.
- Can relate equal lengths of unlike units.
- Can be conceptually understood through diagram literacy.

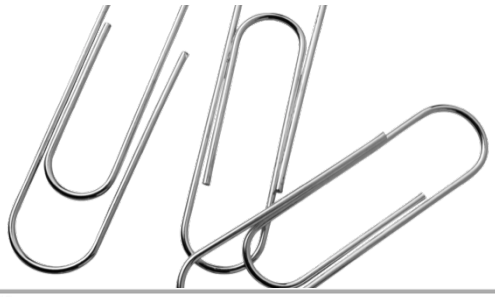
Classroom Application



4th Grade NAEP Item

- A fourth grade class needs five leaves each day to feed its two caterpillars.
- How many leaves would it take to feed twelve caterpillars?
- Solve this problem without cross multiplication.

Paper Clip Chains - AIMS Activity



Helping the Process

- Strong instruction will help students to find the strategies that will involve the fewest computations.
- Students who feel comfortable with a variety of solution strategies will increase their conceptual understanding, as well as their procedural fluency with proportions.

Explore Proportions through Similarity

–Enlargements
–Reductions
–Scale factors
–Indirect Measurement

Students Need Opportunities to…

- Reason about proportional situations
- Find unit rates
- Construct equivalent ratios
- Link ratios, percent and fractions

Bridging to Level 3

Requires a strong understanding of the components of proportional reasoning:

- Change between equivalent ratios is multiplicative, not additive
- The multiplicative change is constant
- The relationship between ratios is the scale factor (If you multiply one ratio by the scale factor, the result is the second ratio.)

What Component of Number Sense can help us explain why cross multiplication works?

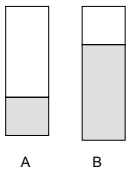
Problems that Encourage Proportional Sense

Adapted from Esther Billings
Mathematical Teaching in the Middle School NCTM

John drove 60 miles in 2 hours. If he continues to drive at this speed, how long will it take him to drive 40 additional miles?

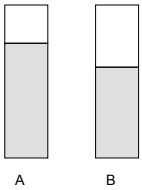
- In the following situations it is indicated which carafe contains the stronger coffee. Determine which carafe will contain the stronger coffee after the alterations have been made.
- Explain how you came to your answer.

Carafe B contains weaker coffee than Carafe A. Add one spoon of instant coffee to carafe A and one cup of water to carafe B.



Which carafe will have the stronger coffee?

Carafe A and Carafe B contain coffee that tastes the same. Add one spoon of instant coffee to both carafe A and B.

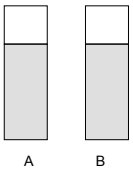


Which carafe will have the stronger coffee?

Trashketball and Carafes?

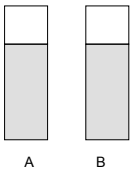
- Trashketball is a proportion too…
- How would you use student knowledge of Trashketball to support their understanding of the carafe situation?

Carafe A contains weaker coffee than Carafe B. Add one spoon of instant coffee to carafe A and carafe B.



Which carafe will have the stronger coffee?

Carafe B contains the stronger coffee. Add one spoon of instant coffee to Carafe A and 1 cup of water to Carafe B.



Which carafe will have the stronger coffee?

Exposing students to nonnumeric quantities will strengthen their proportional reasoning.

Why don't kids think proportionally?

- Because we don't build on their intuitive thinking.
- Because it's hard for them.
- Because we don't teach it very well.
- Because we don't make connections between different forms of numbers.
- Because it's hard for us…

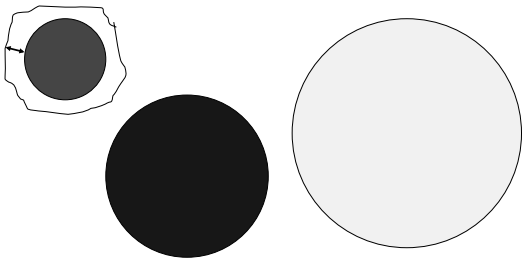
Perimeter around the earth

Imagine there is a string that goes around the earth and represents the earth's perimeter at it's widest arc.

Now increase that string by 1 meter.

How far away would the string now be from the earth's surface?

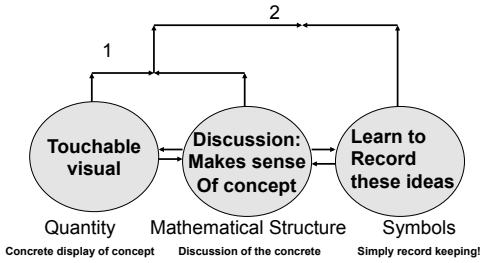
Let's use visuals to help us think this through…



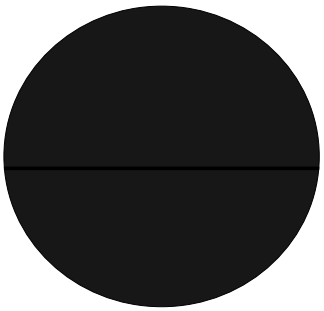
At Your Tables---

- Use the string to represent the circumference of your circle
- Add 1 meter (100 cm) of string to the circumference
- Cut your string. The length of your string represents the circumference of your circle plus 100 cm.
- Use your string to make a larger circle around your original circle. (Both circles should have the same center.)
- Measure and record the distance from the edge of the new circle to the edge of the original circle. (in cm)
- About how much of a meter does this amount represent?

Prototype for lesson construction



V. Faulkner and DPI Task Force adapted from Griffin



$$\frac{c}{d}$$

Circle Relationships

If given d
$d \bullet \pi = C$

If given r
$(2 \bullet r) \bullet \pi = C$

If given C
$C \div \pi = d$
$C \div \pi \bullet 2 = r$

Questioning Techniques: Developing Number Sense

If a circle has a circumference of 12, about what is it's diameter?

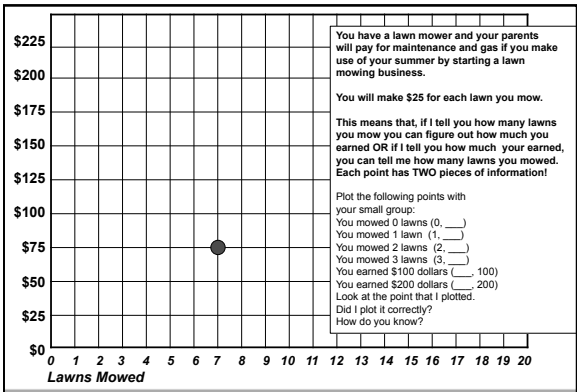
If a circle has a diameter of 10, about what is its circumference?

If a circle has a circumference of 32, about what is its diameter? It's radius?

If a circle has a radius of 4, about what is its circumference?

Proportional Thinking…

Let's reinforce what we know about proportional relationships to make sure we are ready to reason with our string around the earth problem…



You have a lawn mower and your parents will pay for maintenance and gas if you make use of your summer by starting a lawn mowing business.

You will make $25 for each lawn you mow.

This means that, if I tell you how many lawns you mow you can figure out how much you earned OR if I tell you how much your earned, you can tell me how many lawns you mowed. Each point has TWO pieces of information!

Plot the following points with your small group:
You mowed 0 lawns (0, ___)
You mowed 1 lawn (1, ___)
You mowed 2 lawns (2, ___)
You mowed 3 lawns (3, ___)
You earned $100 dollars (___, 100)
You earned $200 dollars (___, 200)
Look at the point that I plotted.
Did I plot it correctly?
How do you know?

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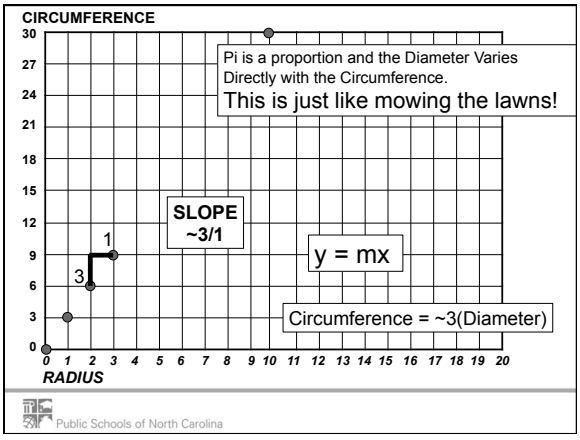
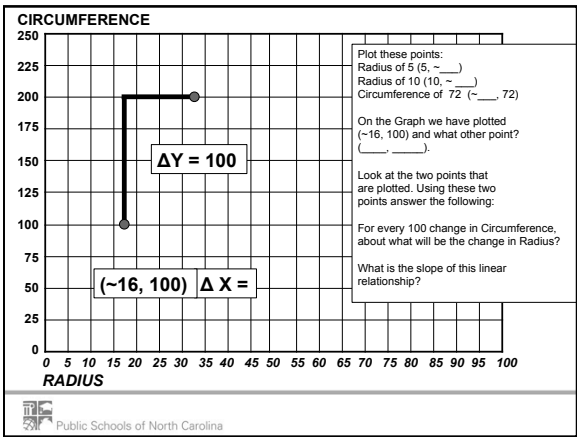
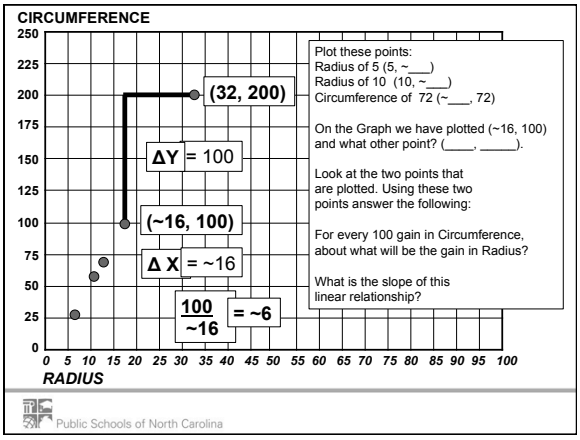
Now, back to the earth---






CIRCUMFERENCE
250
225
200
175
150
125
100
75
50
25
0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
RADIUS
(32, 200)
ΔY = 100
(~16, 100)
Δ X = ~16
100 / ~16 = ~6
Plot these points:
Radius of 5 (5, ~___)
Radius of 10 (10, ~___)
Circumference of 72 (~___, 72)
On the Graph we have plotted (~16, 100) and what other point? (____, _____).
Look at the two points that are plotted. Using these two points answer the following:
For every 100 gain in Circumference, about what will be the gain in Radius?
What is the slope of this linear relationship?
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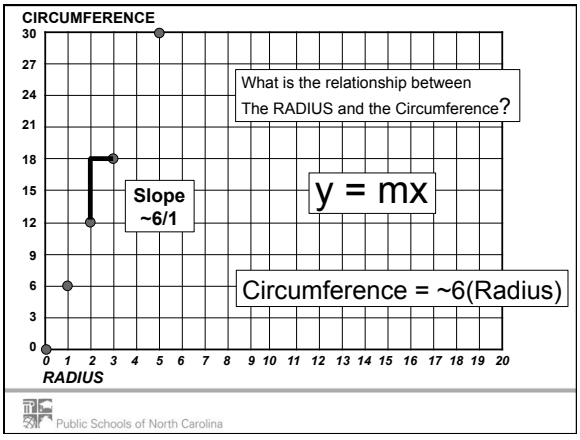
CIRCUMFERENCE
30
27
24
21
18
15
12
9
6
3
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
RADIUS
What is the relationship between The RADIUS and the Circumference?
Slope ~6/1
y = mx
Circumference = ~6(Radius)
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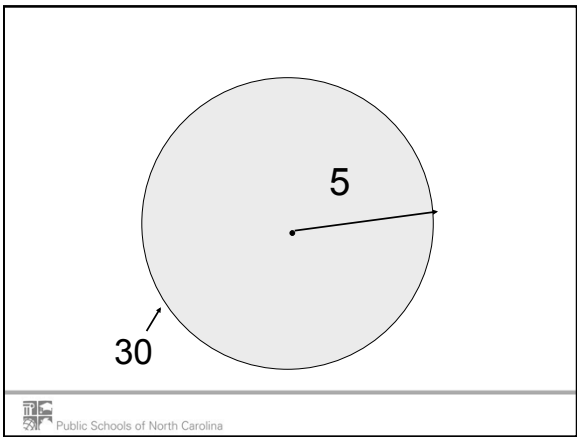
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30
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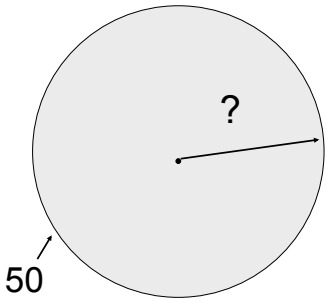
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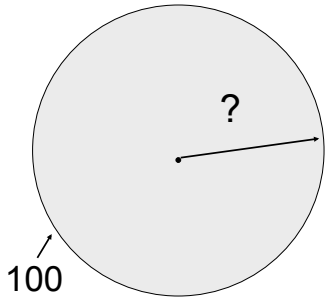
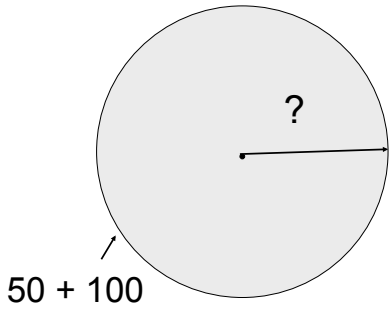
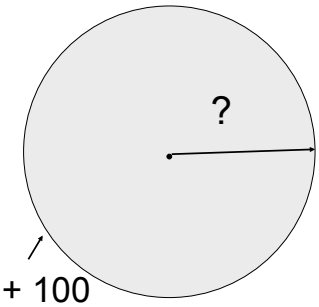
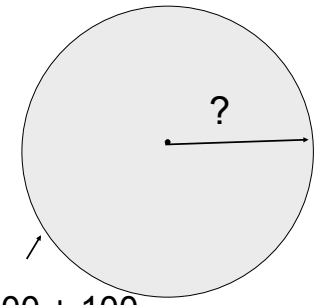

?
100


?
50 + 100

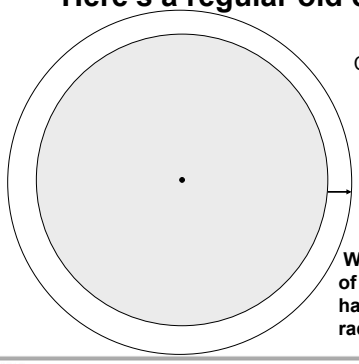


100 + 100



1,000,000 + 100

Here's a regular old circle…

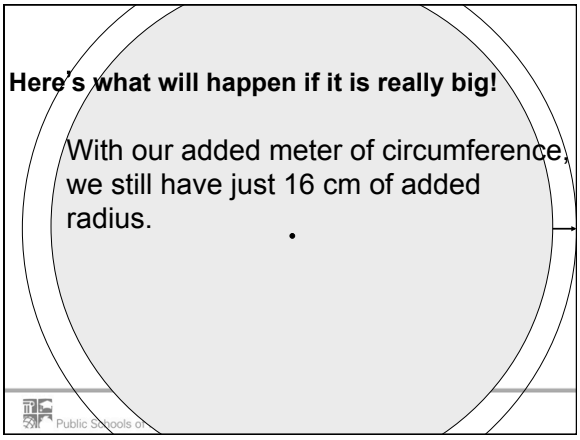


Circumference is 50 cm

With our added meter of circumference we have 16 cm of added radius.

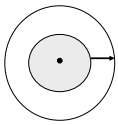
Here's what will happen if it is really big!

With our added meter of circumference, we still have just 16 cm of added radius.



What if it gets really small?

Still, when we add a meter of circumference we still have just 16 cm of added radius.



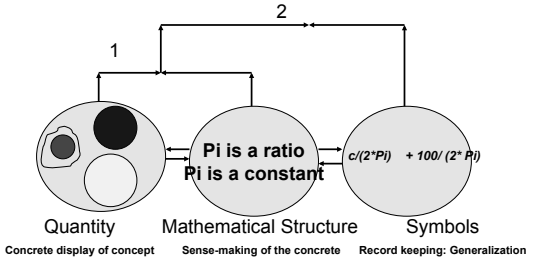
And what if we engage in a 'thought experiment'?

What if our circle goes down to a circumference of 0?

What if we have a circumference of 0?

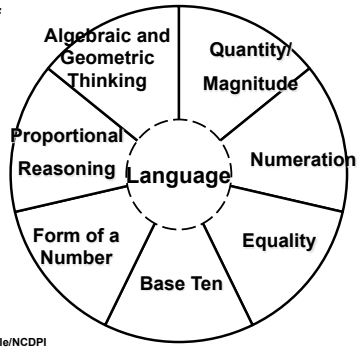
With, a circumference of 0 we'd have <u>no circle at all.</u> We can still add our meter of string to nothing. It will go around a point and we'd STILL be 16 cm away from our center, and have a radius of ~16cm

Pi and the earth



V. Faulkner and DPI Task Force adapted from Griffin

The Components of Number Sense



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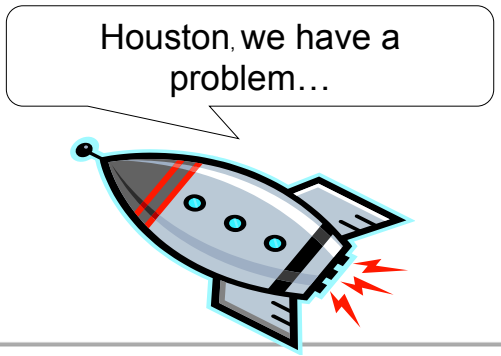
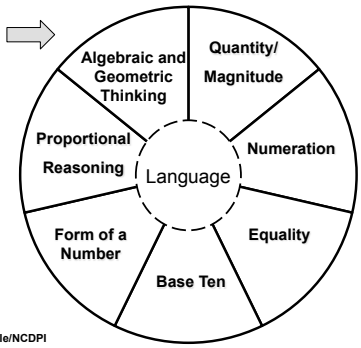


Public Schools of North Carolina

Algebraic and Geometric Thinking

The Components of Number Sense



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Public Schools of North Carolina

Public Schools of North Carolina

Algebraic Thinking

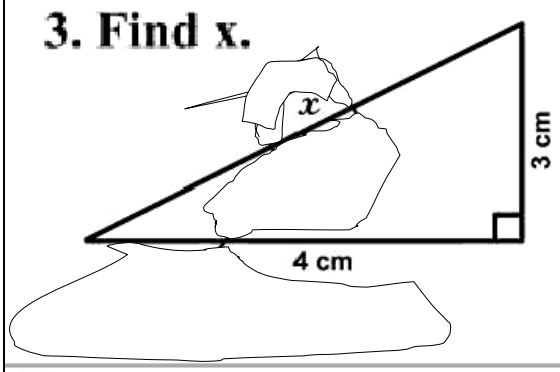
Defining the Concept

Defining the Concept	Diagnosis	Where the Research Meets the Road	Classroom Application

Defining the Concept

The study of the letter x?

3. Find x.



Think about algebra as---

a) a systematic way of expressing generality and abstraction
b) the syntactically guided transformation of symbols.

(Adding It Up, National Research Council)

In simpler terms, algebra is

- Using what we know to find out what we don't know.
- Analyzing patterns and relationships.
- Making predictions, modeling situations, and solving problems.

Verbal:

You and your friends want to record a CD. The company you want to hire charges $300 for making the master and designing the art for the cover. It charges $2.50 for burning each CD. Use the function rule P(c)=300 + 2.5c. Make a table of values and a graph.

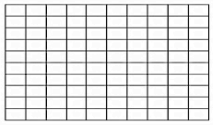
Algebraic

_____

Table

(x)	Function: _____	_____ (y)

Graph



Diagnosis

Defining the Concept	**Diagnosis**	Where the Research Meets the Road	Classroom Application

From Elementary School Arithmetic, a student brings---

- A desire to execute operations rather than use them to represent relationships.
- The use of an equal sign to announce a result rather than to show equality.
- A use of inverse or undoing operations to solve a problems.
- A perspective of letters as representing unknowns but not variables.

Challenges Students Experience with Algebra

- **Translate word problems into mathematical symbols**
- **Describe, paraphrase, or explain**
- **Link the concrete to a representational to the abstract**
- **Remember vocabulary and processes**
- **Show fluency with basic number operations**
- **Maintain focus**
- **Show written work**

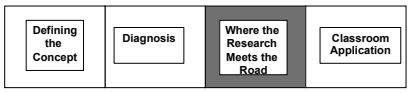
At the Elementary Level, Students with Disabilities Have Difficulty with:

- **Solving problems** **(Montague, 1997; Xin Yan & Jitendra, 1999)**
- **Visually representing problems (Montague, 2005)**
- **Processing problem information** **(Montague, 2005)**
- **Memory** **(Kroesbergen & Van Luit, 2003)**
- **Self-monitoring** **(Montague, 2005)**

At the Middle School Level, Students with Disabilities Have Difficulty:

- **Meeting content standards and passing state assessments** (Thurlow, Albus, Spicuzza, & Thompson, 1998; Thurlow, Moen, & Wiley, 2005)
- **Mastering basic skills** (Algozzine, O'Shea, Crews, & Stoddard, 1987; Cawley, Baker-Kroczynski, & Urban, 1992)
- **Reasoning algebraically** (Maccini, McNaughton, & Ruhl, 1999)
- **Solving problems** (Hutchinson, 1993; Montague, Bos, & Doucette, 1991)

Where the Research Meets the Road



NAEP Task

Give the value of y when x = 3.

x	1	3	4	7	n
y	8		11	14	

3X +7 = Y

- Write a story problem for this equation…

How would your students do?

7th grade?
8th grade?
Algebra 1?
Algebra 2?
Calculus?

Algebra is --

- A gateway to higher levels of mathematics education.
- Abstract thinking required for our full participation in our democratic society and technological driven world.

Dudley and Riley 1998

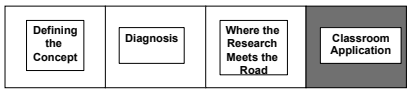
"For students to meaningfully utilize algebra, it is essential that instruction focus on sense making, not symbol manipulation. Throughout their mathematical careers, students should have opportunities to reflect on and talk about general procedures performed on numbers and quantities."

Battista and Brown (1998)

Concrete to Representational to Abstract

- Algebra is fundamentally very concrete!
- Like all math, Algebra is just a way to model concrete realities, actions and situations.
- Why do we bypass the concrete reality of algebra and go directly to symbolic manipulation?

Classroom Application



Relating Quantities

- Teaching Experience,
  Number of Years you plan to continue to teach
  (Red Dots)
- Number of letters in your first name,
  Number of First Cousins
  (Green Dots)
- Distance in miles you drove,
  Time traveled in Minutes
  (Blue Dots)

Relating Quantities

Children make natural connections between numbers.

Children need

- many opportunities to investigate relationships
- visuals provided as numerical data is compared
- non-examples to help set the limits of a concept and to fully structure their understanding.

What does the relationship mean?

- How are data related or not related?
- What new data might come in at a later time?
- How could you use the relationship to predict another data point?
- What could have happened if?
- What happens between data points?
- What is the essence of this pattern?

Other Examples

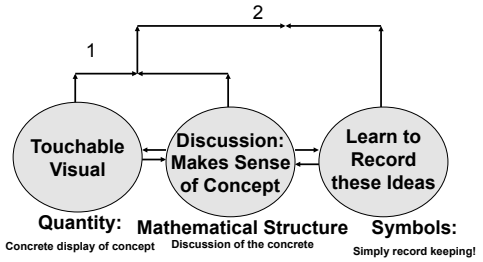
- Number of Pets , Number of family members
- Grade level, Average age
- Number of miles driven, gas in tank
- Age/Grade on Report Card in Math
- Length of foot, height
- Amount of trouble you get into, parent happiness factor

Ordered Pairs can have Predictive Value--

But they do not always represent a Cause and Effect

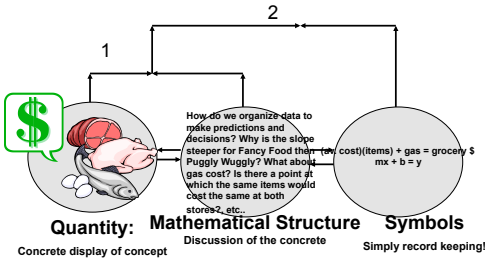
Example: Vocabulary, Height

Prototype for Lesson Construction



V. Faulkner and DPI Task Force adapted from Griffin

Even Algebra can fit this mold!



Faulkner adapting Leinwald, Griffin

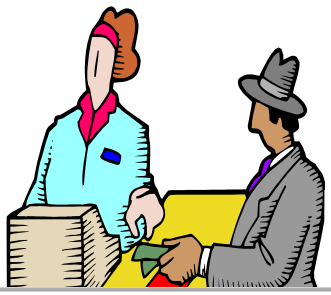
Common Core Standards: Algebra

- Seeing **Structures** in Expressions
- **Creating** Equations
- **Reasoning** with Equations

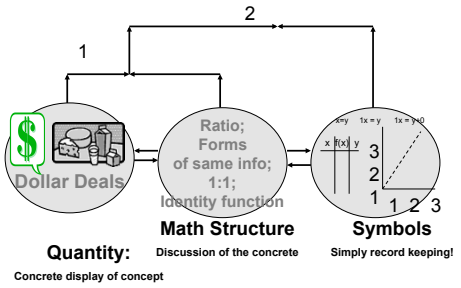
Algebra through Shopping

- The following lesson units/materials are all copyrighted with Wake County Public Schools/Valerie Faulkner.
- These have also been incorporated into Walch Educational publications, Intro to High School Math and Foundations of Mathematics
- Further training in these units is available through DPI.
- The reality that shopping can be modeled with algebra is, of course, public domain.
- Steve Leinwand also presents ideas using average cost from shopping register tickets and speeding ticket formulas to develop algebraic concepts.

Unit 1
Concept Development Dollar Deals



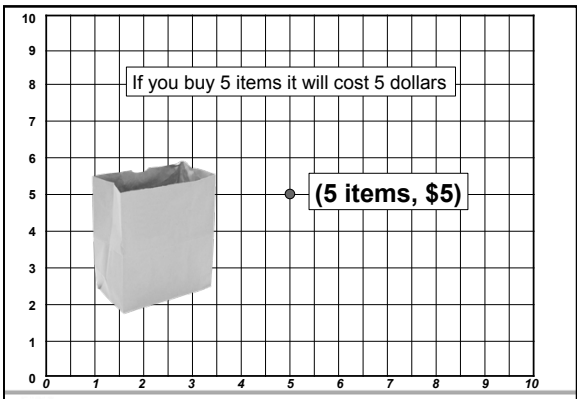
Dollar Deals Learning Objectives



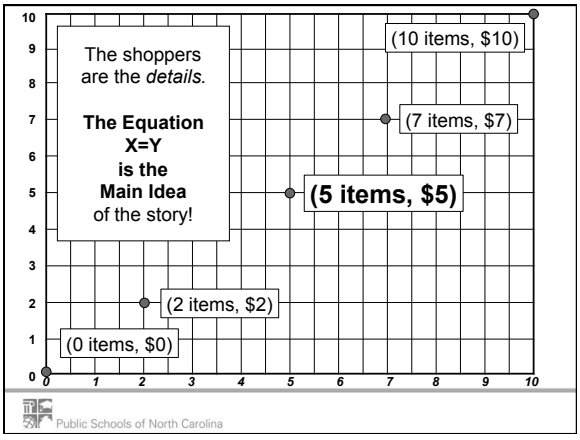
Dollar Deals

No Tax, No Tricks: One Dollar per One Item.

Where EVERYTHING is just one dollar.

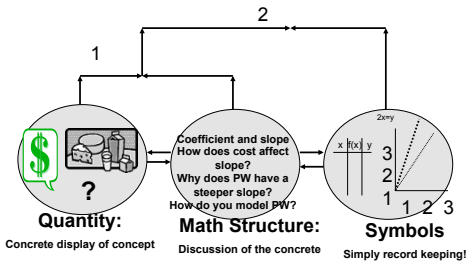


X = Y



Unit 2
Dollar Deals vs. Puggly Wuggly

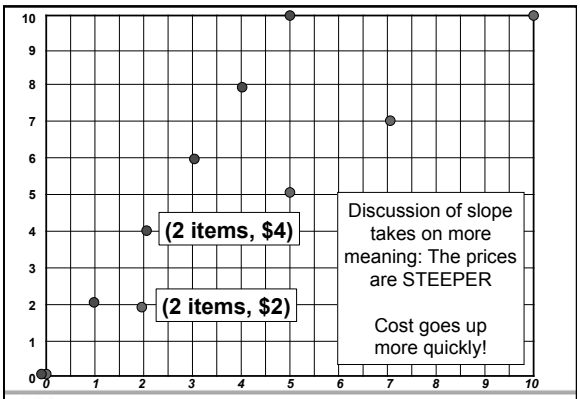
Puggly Wuggly vs. Dollar Deals
Learning Objectives



Puggly Wuggly Super Sale!

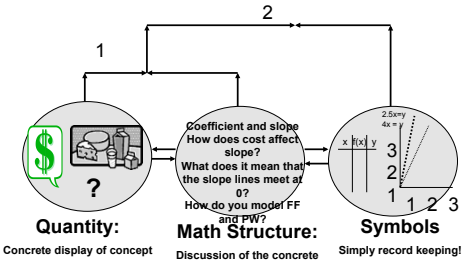
Bag day sale!

All items you can fit in 1 bag:
2 dollars per item!



Unit 3
Puggly Wuggly vs. Fancy Food

Puggly Wuggly vs. Fancy Foods
Learning Objectives



Quantity:
Concrete display of concept

Math Structure:
Discussion of the concrete

Symbols
Simply record keeping!

Puggly Wuggly

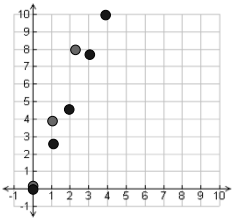
No sale—just our usual price:
$2.50 per item

Fancy Foods

You can pay now, or you can pay later…
The best food at the worst prices!
All organic food
$4 per item

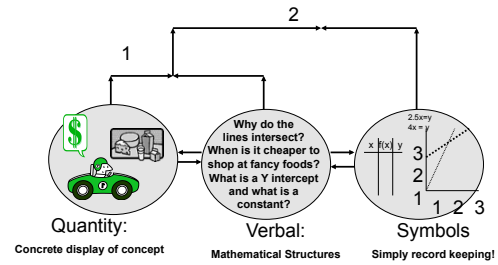
Other Forms of the Number Relationship!

X (Items)	Y=2.5x PW	Y=4x FF
1	2.5	4
2	5	8
3	7.5	12
4	10	16
5	12.5	20

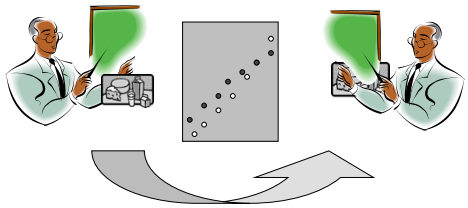


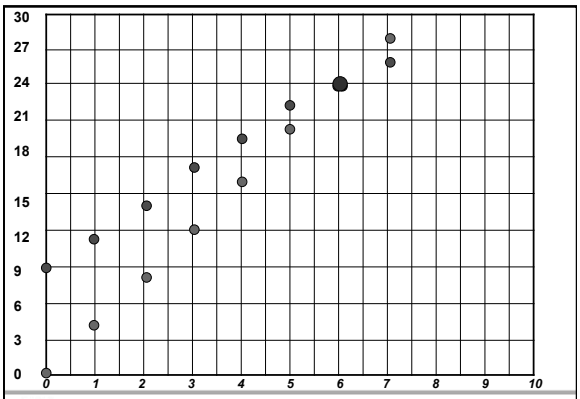
Unit 4
Feeding the DumDee

Feeding the DumDee
Learning Objectives



Feeding the DumDee
What Does it Look Like?





Feeding the DumDee—Big Ideas

Students will begin to see what y = mx + b means in its entirety. Up until now our b has been the constant "0" and so it has "dropped out" of the equation. But now we have a constant other than 0.

How does this affect our equation?

Our t-chart?

Our graph?

Our cost?

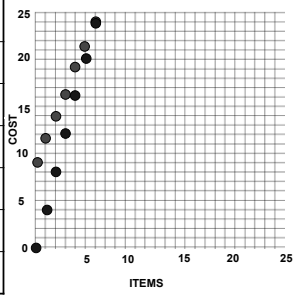
Mathematical Models
The Showdown:
PW + Gas vs. FF on foot

_____

_____

Fancy Food vs. Puggly Wuggly + Gas!

X Items	Y=4x FF	Y=2.50x + 9 PW
1	$4.00	$11.50
2	$8.00	$14.00
3	$12.00	$16.50
4	$16.00	$19.00
5	$20.00	$21.50
6	$24.00	$24.00



Two shoppers, One purpose

Jenn and Jermaine had a little argument about shopping. Jenn said that she could shop for things cheaper than Jermaine, because "guys don't know how to shop." Jermaine thought that was ridiculous and challenged her to a shopping match. "You're on" she said. "Great" said Jermaine, "Save your receipt and you need to include the cost of travel to get to the store." Jenn agreed to these conditions and they were ready to go.

Jenn had an energy efficient hybrid car, so she felt confident that she would win, because Jermaine drove a truck. Puggly Wuggly was far away, but she thought the cheap prices at Puggly's $2/item bag day sale would be worth the drive. She spent $2.00 on gas for the whole trip.

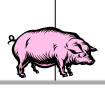
Jermaine knew about the Puggly Bag Sale too. But he didn't drive his truck. He took the bus and it cost him $1.50.

Model the contest between the two with linear equations and then graph them. Answer the questions and be prepared to discuss with the class.

Mathematical Models

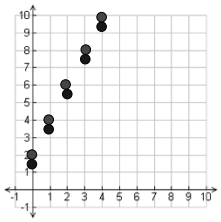
_____

_____

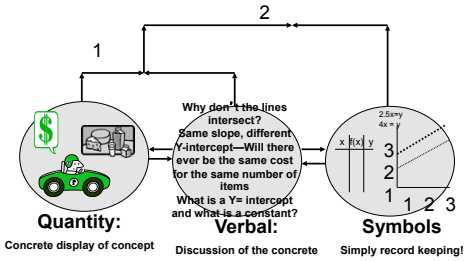
x Number of Items	Rule Math Model	y Cost	x Number of Items	Rule Math Model	y Cost
	+			+	

Jenn and Jermaine's Shopping Trip

X Items	Y=2x + 1.5 Jermaine	Y=2x + 2 Jenn
0	$1.50	$2.00
1	$3.50	$4.00
2	$5.50	$6.00
3	$7.50	$8.00
4	$9.50	$10.00



Feeding the DumDee--Jenn V. Jermaine Learning Objectives



Given two points, find the equation of the line…

$$\frac{Y_1 - Y_2}{X_1 - X_2} \qquad \frac{\triangle Y}{\triangle X}$$

How are your students processing this?

Why the Y?

$$\frac{\text{Cost}}{\text{Item}}$$

Cost ***per*** Item

$$\frac{\triangle \text{ Dependent}}{\triangle \text{ Independent}} \qquad \frac{\triangle \text{ Y}}{\triangle \text{ X}}$$

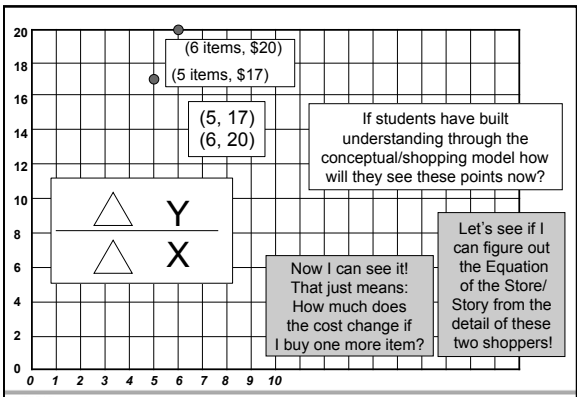
How can I get from here:

(5,17)
(6, 20)

to here:

3x + 2 = y

with MEANING?



- Seeing **Structures**
- **Creating** Equations
- **Reasoning** with Equations

Verbal:

You and your friends want to record a CD. The company you want to hire charges $300 for making the master and designing the art for the cover. It charges $2.50 for burning each CD. Use the function rule P(c)=300 + 2.5c. Make a table of values and a graph.

Algebraic

_____

Table

_____ (x)	Function: _____	_____ (y)

Graph

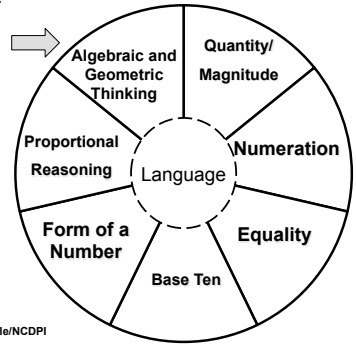
Assignments from Units 7

Learning Task 7 –Painting Walls

Required Reading for Unit 9
(Be prepared to discuss in your groups.
"Mathematics Education in the 21st Century"
Deborah Ball

Geometric Thinking

The Components of Number Sense



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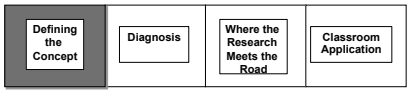
Geometric Thinking

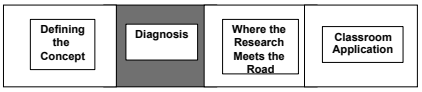
Defining the Concept



Geometric Thinking Involves ---

- the study of geometric figures (shape and form) and their properties.
- congruence, symmetry, similarity and transformations.
- spatial measure.

Diagnosis



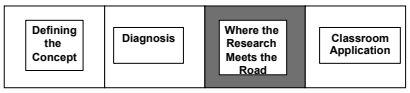
Levels of Geometric Thinking

van Hiele Levels

- Level 0 Visual
- Level 1 Analysis
- Level 2 Informal Deduction
- Level 3 Deduction
- Level 4 Rigor

Van de Walle, John A. (2001) Geometric Thinking and Geometric Concepts

Where the research meets the road

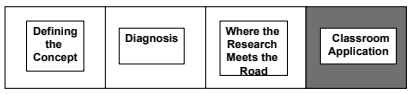


Children enter school with a sense of informal knowledge of geometry.

"Instruction in Geometry needs to complement the study of number and operations in Grades Pre-K through 8."

Adding It Up ***National Research Council***

Classroom Application:



Foundation Level Training

Understanding the Relationship between Area and Perimeter

Using Griffin's Model

Mayer's Problem Solving Model

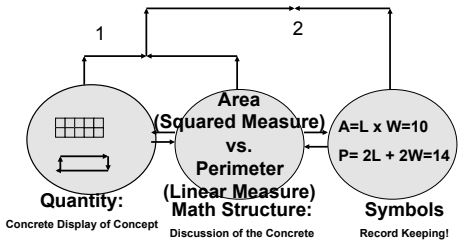
Components of Number Sense

Problem

MAX

- What are the whole number dimensions of a rectangle that can produce the greatest area given 150 feet of fencing?
- Context: There is a leash law at Atlantic Beach. What is the largest rectangular pen that could be built for Max with 150 feet of fencing for the times he is not on his leash?

Prototype for Lesson Construction



V. Faulkner and DPI Task Force adapted from Griffin

Cognitive Processes in Problem Solving Richard E. Mayer

- Translating
- Integrating
- Planning
- Executing

Cognitive Processes in Problem Solving
Richard E. Mayer

Translating

Frayer Model for Linguistics

Whole Numbers

Definition
(in own words)

Counting Numbers plus 0

Characteristics

- **Rational**
- **Positive, except for 0**

Examples

0, 1, 2, 3, 4, …

Non-Examples

$\sqrt{2}$ $\frac{1}{2}$

-1, -2, ,-3 …;

Frayer Model for Linguistics

Perimeter

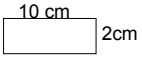
Definition
(in own words)

The distance around an object (linear measure)

Characteristics

- Involves measuring in one dimension
- Involves computing with addition
- Involves problem solving

Examples



Perimeter is 24 cm

Non-Examples

Area is 20 sq. cm².

Frayer Model for Linguistics

Definition
(in own words)

The measure of the interior of a figure

(squared measure)

Characteristics

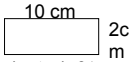
- Involves measuring in two dimensions
- Involves computing with multiplication
- Involves problem solving

Area

Examples

Area is 20 cm²

Non-Examples



Perimeter is 24 cm.

Problem Solving-Translating

- I have 150 feet of fencing. When I use fencing, it will go around the distance of a figure (perimeter), in this case a rectangle.
- I know that rectangles have two pairs of opposite congruent sides.
- I know that when I am finding perimeter, I compute with addends.
- I know I can only use whole numbers and 0 cannot be the measure of a side.
- I know that there are many different pairs of sides that I can create with 150 feet of fencing.

Problem Solving-Translating

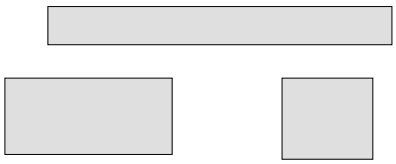
- I know area is the squared measure of the interior of a figure.
- I know that the largest area is the figure whose interior measure is the greatest.
- I know that area can be represented by an array and I can use repeated addition or multiplication to solve for area.

Cognitive Processes in Problem Solving
Richard E. Mayer

Integrating

Problem Solving-Integrating

Building a mental model of the problem.



If the Perimeter is 150 feet,
which rectangle will have the greatest area?

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Planning

Let's Look at An Easier Problem

- Cafeteria Tables and Students
- Bag of 30 tiles

Cafeteria Tables

A one unit table will be able to seat four students.

A two unit table will be able to seat six students.

Following the pattern---

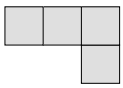
A three unit table will sit eight students.

---even with this arrangement.

But what about---

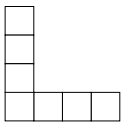
A four unit table? It can seat ten students.

---even when arranged like this.



But what about this arrangement?

Challenge 1



Area = 7 **P =** 16

What other perimeter could be shown that has the same area?

Challenge 2

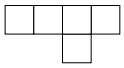
A = 5 **P =** 12

Rearrange the area to show a diagram with a different perimeter.

Challenge 3

Add a square so that the area is 6 and the perimeter remains 12.

A = 5 **P =** 12

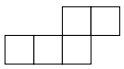


Can you find other arrangements with an area of 6 and a perimeter of 12?

Challenge 4

Add squares so that the perimeter is 18.
What is the new area?

A = 5 **P =** 12



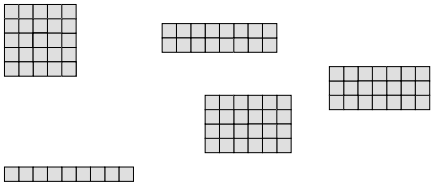
What other arrangements could be made to create different areas with the same perimeter?

Looking at a Simpler Problem

Problem: What is the greatest rectangular area that can be produced with 20 feet of fencing?

Problem Solving-Planning

Concrete- All Rectangles with a Perimeter of 20 with whole number dimensions.



Problem Solving-Planning

Representational/Verbal Discussion

- How can we organize the data?
- Which rectangle has the greatest area given the perimeter?
- What do you notice about the area of the rectangle as the sides increase or decrease?
- What rule can we find that will help us to find the greatest area of a rectangle with a given perimeter?
- How can we use the easier problem to find a pattern for the more difficult problem?

How Can We Organize the Data?

Length	Width	P= 2L + 2 W	A = L x W
1	9	P= 20 units	A = 9 sq. units
2	8	P= 20 units	A = 16 sq. units
3	7	P= 20 units	A = 21 sq. units
4	6	P= 20 units	A = 24 sq. units
5	5	P= 20 units	A = 25 sq. units
6	4	P= 20units	A = 24 sq. units
7	3	P= 20units	A = 21 sq. units
8	2	P= 20units	A = 16 sq. units
9	1	P= 20 units	A = 9 sq. units

Cognitive Processes in Problem Solving
Richard E. Mayer

Executing

Problem Solving-Executing

Symbolic

- Rectangle Dimensions
  - 1 x 9 = 9 square units
  - 2 x 8 = 16 square units
  - 3 x 7 = 21 square units
  - 4 x 6 = 24 square units
  - 5 x 5 = 25 square units (largest area!!!)

So what about---

150 feet of fencing?

Which Rectangle Will Yield the Greatest Area?

- A 1 foot by 74 foot rectangle?
- A 2 foot by 73 foot rectangle?
- ……etc.
- A 37 x 38 rectangle-----

Will Make Max the Happiest!

Questioning through the Components of Number Sense

- Quantity & Magnitude
- Numeration
- Equality
- Base Ten
- Form of a Number
- Proportional Reasoning
- Algebraic and Geometric Thinking

Foot Stuff –

Marilyn Burns

Exploring Area and Perimeter

Area and Perimeter

- Draw a line around your bare foot on the graph paper.
- Figure the area of your foot and record the result inside the foot outline. (in square units)
- Place the string on foot outline and cut to same length (perimeter).
- Take your foot perimeter string and shape it into a square on the graph paper.
- Draw the outline (perimeter) of the square.
- Figure the area of that square and record the result inside the string square. (in square units)

Area and Perimeter

- Are perimeters the same for both shapes?
- Are areas the same for both shapes?

Clarifying the Conditions

- Three Cases
  - The length <u>or</u> width is increased
  - The length <u>and</u> width are increased
  - <u>Either</u> the length <u>or</u> width <u>increases</u> while the other dimension <u>decreases</u>

Foot Stuff Questions

- What method did you use to figure the area of your foot?
- How did you figure out how to arrange your perimeter string into the shape of a square?
- How does the area of your foot compare to the area of the string square?
- Why do you think that this makes sense that the area of your foot and string square are different?

Area & Perimeter

- How well do we/our students visualize a distance measurement vs. a squared measurement?
- How do we develop these concepts for our students?
- How do we prepare students for volume (cubed measurements)?

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